Imaging, Deconvolution & Image Analysis
I. Theory

Jérôme PETY
(IRAM/Obs. de Paris)

7th IRAM Millimeter Interferometry School
Oct. 4 - Oct. 8 2010, Grenoble
Scientific Analysis of a mm Interferometer Output

mm interferometer output:
  Calibrated visibilities in the $uv$ plane ($\simeq$ the Fourier plane).

2 possibilities:
  • $uv$ plane analysis (cf. Lecture by A. Castro-Carrizo):
    Always better . . . when possible!
    (in practice for “simple” sources as point sources or disks)
  • Image plane analysis:
    $\Rightarrow$ Mathematical transforms to go from $uv$ to image plane!

Goal: Understand effects of the imaging process on
  • The resolution;
  • The field of view (single pointing or mosaicing, cf. Lecture by F. Gueth);
  • The reliability of the image;
  • The noise level and repartition (cf. lecture by S. Guilloteau).
From Calibrated Visibilities to Images:
I. Comparison Visibilities/Source Fourier Transform

\[ V_{ij}(b_{ij}) = 2D \text{ FT} \left\{ B_{\text{primary}}.I_{\text{source}} \right\}(b_{ij}) + N \]

- Primary Beam
  \( \Rightarrow \) Distorted source information.
- Noise \( \Rightarrow \) Sensitivity problems.
- Irregular, limited sampling
  \( \Rightarrow \) incomplete source information:
  - Support limited at:
    * High spatial frequency
      \( \Rightarrow \) limited resolution;
    * Low spatial frequency \( \Rightarrow \) problem of wide field imaging;
  - Inside the support, incomplete (\textit{i.e.} Nyquist's criterion not respected) sampling \( \Rightarrow \) lost of information.

Imaging, Deconvolution & Image Analysis  
J. Pety, 2010
From Calibrated Visibilities to Images:  
II. Effect of Irregular, Limited Sampling

Definitions:

- \( V = \text{2D FT}\{B_{\text{primary}}.I_{\text{source}}\}; \)
- Irregular, limited sampling function:
  - \( S(u, v) = 1 \) at \((u, v)\) points where visibilities are measured;
  - \( S(u, v) = 0 \) elsewhere;
- \( B_{\text{dirty}} = \text{2D FT}^{-1}\{S\}; \)
- \( I_{\text{meas}} = \text{2D FT}^{-1}\{S.V\}. \)

Fourier Transform Property #1:

\[ I_{\text{meas}} = B_{\text{dirty}} \ast \{B_{\text{primary}}.I_{\text{source}}\}. \]

\( B_{\text{dirty}}: \) Point Spread Function (PSF) of the interferometer  
(i.e. if the source is a point, then \( I_{\text{meas}} = I_{\text{tot}}.B_{\text{dirty}} \)).
From Calibrated Visibilities to Images: III. Why Deconvolving?

- Difficult to do science on dirty image.
- Deconvolution $\Rightarrow$ a clean image compatible with the sky intensity distribution.
From Calibrated Visibilities to Images: Summary

Fourier Transform and Deconvolution: The two key issues in imaging.

<table>
<thead>
<tr>
<th>Stage</th>
<th>Implementation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calibrated Visibilities</td>
<td></td>
</tr>
<tr>
<td>↓ Fourier Transform</td>
<td>GO UVSTAT, GO UVMAP</td>
</tr>
<tr>
<td>Dirty beam &amp; image</td>
<td></td>
</tr>
<tr>
<td>↓ Deconvolution</td>
<td>GO CLEAN</td>
</tr>
<tr>
<td>Clean beam &amp; image</td>
<td></td>
</tr>
<tr>
<td>↓ Visualization</td>
<td>GO BIT, GO VIEW</td>
</tr>
<tr>
<td>↓ Image analysis</td>
<td>GO NOISE, GO FLUX, GO MOMENTS</td>
</tr>
<tr>
<td>Physical information on your source</td>
<td></td>
</tr>
</tbody>
</table>

Imaging, Deconvolution & Image Analysis J. Pety, 2010
From Calibrated Visibilities to Images: Summary

Fourier Transform and Deconvolution:
The two key issues in imaging.

<table>
<thead>
<tr>
<th>Stage</th>
<th>Implementation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calibrated Visibilities</td>
<td></td>
</tr>
<tr>
<td>⇓ Fourier Transform</td>
<td>GO UVSTAT, GO UVMAP</td>
</tr>
<tr>
<td>Dirty beam &amp; image</td>
<td></td>
</tr>
<tr>
<td>⇓ Deconvolution</td>
<td>GO CLEAN</td>
</tr>
<tr>
<td>Clean beam &amp; image</td>
<td></td>
</tr>
<tr>
<td>⇓ Visualization</td>
<td>GO BIT, GO VIEW</td>
</tr>
<tr>
<td>⇓ Image analysis</td>
<td>GO NOISE, GO FLUX, GO MOMENTS</td>
</tr>
<tr>
<td>Physical information on your source</td>
<td></td>
</tr>
</tbody>
</table>
Direct vs. Fast Fourier Transform

Direct FT:
- Advantage: Direct use of the irregular sampling;
- Inconvenient: Slow.

Fast FT:
- Inconvenient: Needs a regular sampling $\Rightarrow$ Gridding;
- Advantage: Quick for images of size $2^M \times 2^N$.

$\Rightarrow$ In practice, everybody use FFT.
Gridding: I. Interpolation Scheme

Convolution because:

- **Visibilities** = noisy samples of a smooth function.
  \[ \Rightarrow \text{Some smoothing is desirable.} \]
- Nearby visibilities are not independent.
  \[ V = \text{2D FT} \left\{ B_{\text{primary}}, I_{\text{source}} \right\} = \tilde{B}_{\text{primary}} \ast \tilde{I}_{\text{source}}; \]
  \[ \text{FWHM(} \text{convolution kernel} \text{)} < \text{FWHM}(\tilde{B}_{\text{primary}}) \]
  \[ \Rightarrow \text{No real information lost.} \]
Gridding: II. Convolution Equation is Kept Through Gridding

Demonstration:

• \( I_{\text{meas}}^{\text{grid}} \xrightarrow{\text{2D FT}} G \ast (S.V) \Leftrightarrow I_{\text{meas}}^{\text{grid}} = \tilde{G} \ast (\tilde{S} \ast \tilde{V}); \)

• \( B_{\text{dirty}}^{\text{grid}} \xrightarrow{\text{2D FT}} G \ast S \Leftrightarrow B_{\text{dirty}}^{\text{grid}} = \tilde{G} \ast \tilde{S}; \)

\( \Rightarrow I_{\text{meas}} = B_{\text{dirty}} \ast \left\{ B_{\text{primary}} \ast I_{\text{source}} \right\} \)

with \( I_{\text{meas}} = I_{\text{meas}}^{\text{grid}} / \tilde{G} \)

and \( B_{\text{dirty}} = B_{\text{dirty}}^{\text{grid}} / \tilde{G}. \)

Remark: Gridding may be hidden in equations but it is still there.

\( \Rightarrow \) Artifacts due to gridding! (cf. next transparencies)
Gridding:
III. Effect of a Regular Sampling (Periodic Replication)

**uv Plane**

- $f(x)$
- $\mathcal{F}\left\{ \frac{x}{\tau} \right\}$
- $\mathcal{F}\left\{ \frac{x}{\tau} \right\} f(x)$
- $\frac{1}{2s} \left\{ \frac{x}{\tau} \right\} f(x)$

**Image Plane**

- $F(s)$
- $\tau \mathcal{F}\left\{ \frac{s}{\tau s} \right\}$
- $\tau \mathcal{F}\left\{ \frac{s}{\tau s} \right\} * F(s)$
- $(2s)^{-1} \mathcal{F}\left\{ \frac{s}{2s} \right\} * F(s)$$

**$B_{primary} \cdot I_{source}$**

Regular Sampling function

Result for a fine sampling

Result for critical sampling (Nyquist’s criterion)

Result for a coarse sampling

*Imaging, Deconvolution & Image Analysis*  
J. Pety, 2010
Gridding: III. Effect of a Regular Sampling (Aliasing)

Aliasing = Folding of intensity outside the image size into the image.
⇒ Image size must be large enough.
Pixel size: Between 1/4 and 1/5 of the synthesized beam size (i.e. more than the Nyquist’s criterion in image plane to ease deconvolution).

Image size:

- \( = \) uv plane sampling rate (FT property \# 2);
- Natural resolution in the uv plane: \( \tilde{B}_{\text{primary}} \) size;
  \( \Rightarrow \) At least twice the \( B_{\text{primary}} \) size (i.e. Nyquist’s criterion in uv plane).
Bright Sources in $B_{\text{primary}}$ sidelobes outside image size will be aliased into image.

⇒ Spurious source in your image!

Solution: Increase the image size.

(Be careful: only when needed for efficiency reasons!)
Gridding: VI. Noise Distribution

- Multiplication by 2D TF(G) for interpolation
- Aliasing due to regular sampling in uv plane
- Division by 2D TF(G)
- Division by primary beam
Gridding function must:

- Fall off quickly in image plane (to avoid noise aliasing);
- Fall off quickly in $uv$ plane (to avoid too much smoothing).

$\Rightarrow$ Define a mathematical class of functions: Spheroidal functions.

GILDAS implementation: In GO UVMAP

- Spheroidal functions = Default gridding function;
- Tabulated values are used for speed reasons.
Dirty Beam Shape and Image Quality

\[ B_{\text{dirty}} = 2D \text{ FT}^{-1}\{S\}. \]

Importance of the Dirty Beam Shape:

- Deconvolving a dirty image is a delicate stage;
- The closest to a Gaussian \( B_{\text{dirty}} \) is, the easier the deconvolution;
- Extreme case:
  \[ B_{\text{dirty}} = \text{Gaussian} \Rightarrow \text{No deconvolution needed at all!} \]

Ways to improve (at least change) \( B_{\text{dirty}} \) shape:

- Increase the number of antenna (costly).
- Change the antenna layout (technically difficult).
- Weight the irregular, limited sampling function \( S \) (the only thing you can do in practice).
Dirty Beam Shape and Number of Antenna:
2 Antenna
Dirty Beam Shape and Number of Antenna:

3 Antenna
Dirty Beam Shape and Number of Antenna:

4 Antenna

uv Plane Sampling

Associated Dirty Beam
Dirty Beam Shape and Number of Antenna:

5 Antenna

uv Plane Sampling

Associated Dirty Beam
Dirty Beam Shape and Number of Antenna:

6 Antenna

uv Plane Sampling

Associated Dirty Beam
Dirty Beam Shape and Super Synthesis

uv Plane Sampling

Associated Dirty Beam

Imaging, Deconvolution & Image Analysis

J. Pety, 2010
Dirty Beam Shape and Super Synthesis
Dirty Beam Shape and Super Synthesis

uv Plane Sampling

Associated Dirty Beam

Imaging, Deconvolution & Image Analysis  

J. Pety, 2010
Dirty Beam Shape and Super Synthesis

uv Plane Sampling

Associated Dirty Beam
Dirty Beam Shape and Super Synthesis
Dirty Beam Shape and Super Synthesis

uv Plane Sampling

Associated Dirty Beam
Dirty Beam Shape and Super Synthesis

uv Plane Sampling

Associated Dirty Beam
Dirty Beam Shape and Weighting

Natural Weighting: Default definition of the irregular sampling function at \( uv \) table creation.

- \( S(u, v) = \frac{1}{\sigma^2} \) at \((u, v)\) points where visibilities are measured;
- \( S(u, v) = 0 \) elsewhere;

with \( \sigma^2(u, v) \) the noise variance of the visibility.

Introduction of a weighting function \( W(u, v) \):

- \( B_{\text{dirty}} = 2D \ FT^{-1} \{W.S\} \);
- Robust weighting: \( W \) enhance the large baseline contribution;
- Tapering: \( W \) enhance the small baseline contribution.
Robust Weighting: I. Definition

Definitions:

- Natural = \( \sum_{(u,v)\in \text{Cell}} S \);

- \( \sum_{(u,v)\in \text{Cell}} W.S = \begin{cases} \text{Constant} & \text{if } (\text{Natural} \geq \text{Threshold}); \\ \text{Natural} & \text{else}; \end{cases} \)

- In practice, the cell size is 0.5\( D \) where \( D \) is the single-dish antenna diameter (i.e. 15m for PdBI).
Robust Weighting: II. Examples

Dirty Beam

Natural Weighting

Beam: 1.33 x 0.97

Noise: 0.41 mJy 7.84 mK

Robust Weighting (18)

Beam: 1.26 x 0.93

Noise: 0.41 mJy 8.6 mK

Dirty Image

Clean Image

Imaging, Deconvolution & Image Analysis

J. Pety, 2010
Robust Weighting: II. Examples

Dirty Beam
Natural Weighting
Beam: 1.33 x 0.97
Noise: 0.41 mJy 7.84 mK

Dirty Image
Robust Weighting (10)
Beam: 0.94 x 0.69
Noise: 0.42 mJy 15.8 mK

Clean Image
Robust Weighting: II. Examples

Natural Weighting

Beam: 1.33 x 0.97
Noise: 0.41 mJy 7.84 mK

Robust Weighting (5.62)

Beam: 0.81 x 0.76
Noise: 0.43 mJy 17.24 mK
Robust Weighting: II. Examples

Dirty Beam

Natural Weighting

Beam: 1.33 x 0.97
Noise: 0.41 mJy 7.84 mK

Dirty Image

Robust Weighting (1.78)

Beam: 0.83 x 0.56
Noise: 0.48 mJy 25.5 mK

Clean Image
Robust Weighting: III. Definition and Properties

Definitions:

- Natural = \( \sum_{(u,v) \in \text{Cell}} S \);
- \( \sum_{(u,v) \in \text{Cell}} W.S = \begin{cases} \text{Constant} & \text{if (Natural} \leq \text{Threshold}); \\ \text{Natural} & \text{else}; \end{cases} \)
- In practice, the cell size is 0.5\( D \).

Properties:

- Increase the resolution;
- Lower the sidelobes;
- Degrade point source sensitivity.
Tapering: I Definition

Definition:

- Apodization of the $uv$ coverage in general by a Gaussian;
- $W = \exp \left\{ -\frac{\left( u^2 + v^2 \right)}{t^2} \right\}$ where $t =$ tapering distance.

$\Rightarrow$ Convolution (i.e. smoothing) of the image by a Gaussian.
Tapering: II. Examples

Dirty Beam

Natural Weighting

Beam: 1.33 x 0.97
Noise: 0.41 mJy 7.84 mK

Dirty Image

Tapered Imaging (750 m)

Beam: 1.31 x 1.01
Noise: 0.41 mJy 7.69 mK

Clean Image
Tapering: II. Examples

Natural Weighting

Beam: 1.33 x 0.97
Noise: 0.41 mJy 7.84 mK

Tapered Imaging (340 m)

Beam: 1.4 x 1.17
Noise: 0.42 mJy 6.38 mK

Dirty Beam  |  Dirty Image  |  Clean Image
Tapering: II. Examples

Natural Weighting

Beam: 1.33 x 0.97
Noise: 0.41 mJy 7.84 mK

Tapered Imaging (170 m)

Beam: 1.96 x 1.79
Noise: 0.49 mJy 3.45 mK

Dirty Beam
Dirty Image
Clean Image
Tapering: III. Definition and Properties

Definition:

- Apodization of the $uv$ coverage in general by a Gaussian;

\[ W = \exp \left\{ -\frac{(u^2 + v^2)}{t^2} \right\} \]

where $t = \text{tapering distance}$.

$\Rightarrow$ Convolution (i.e. smoothing) of the image by a Gaussian.

Properties:

- Decrease the resolution;
- Degradate point source sensitivity;
- Increase sensitivity to “medium size” structures.

Inconvenient: Throw out some information.

$\Rightarrow$ To increase sensitivity to extended sources, use compact arrays not tapering.
Weighting and Tapering: Summary

<table>
<thead>
<tr>
<th></th>
<th>Robust</th>
<th>Natural</th>
<th>Tapering</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resolution</td>
<td>High</td>
<td>Medium</td>
<td>Low</td>
</tr>
<tr>
<td>Side Lobes</td>
<td>↘</td>
<td>Medium</td>
<td>？</td>
</tr>
<tr>
<td>Point Source Sensitivity</td>
<td>↘</td>
<td>Maximum</td>
<td>↘</td>
</tr>
<tr>
<td>Extended Source Sensitivity</td>
<td>↘</td>
<td>Medium</td>
<td>↗</td>
</tr>
</tbody>
</table>

Non-circular tapering:
Sometimes ⇒ Better (*i.e.* more circular) beams.
### From Calibrated Visibilities to Images: Summary

Fourier Transform and Deconvolution: The two key issues in imaging.

<table>
<thead>
<tr>
<th>Stage</th>
<th>Implementation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calibrated Visibilities</td>
<td><strong>GO UVSTAT, GO UVMAP</strong></td>
</tr>
<tr>
<td>‡ Fourier Transform</td>
<td></td>
</tr>
<tr>
<td>Dirty beam &amp; image</td>
<td><strong>GO CLEAN</strong></td>
</tr>
<tr>
<td>‡ Deconvolution</td>
<td></td>
</tr>
<tr>
<td>Clean beam &amp; image</td>
<td><strong>GO BIT, GO VIEW</strong></td>
</tr>
<tr>
<td>‡ Visualization</td>
<td></td>
</tr>
<tr>
<td>‡ Image analysis</td>
<td><strong>GO NOISE, GO FLUX, GO MOMENTS</strong></td>
</tr>
<tr>
<td>Physical information</td>
<td></td>
</tr>
<tr>
<td>on your source</td>
<td></td>
</tr>
</tbody>
</table>
Deconvolution: I. Philosophy

\[ I_{\text{meas}} = B_{\text{dirty}} \ast \left\{ B_{\text{primary}} \cdot I_{\text{source}} \right\} + N. \]

Information lost:

- Irregular, incomplete sampling ⇒ convolution by \( B_{\text{dirty}} \);
- Noise ⇒ Low signal structures undetected.

⇒ 1. Impossible to recover the intrinsic source structure!
⇒ 2. Infinite number of solutions!

\( \begin{cases} \text{\( S \) solution (i.e. } I_{\text{meas}} = B_{\text{dirty}} \ast S + N) \\ B_{\text{dirty}} \ast R = 0 \end{cases} \) \( \Rightarrow (S + R) \) solution.
Deconvolution: I. Philosophy (continued)

\[ I_{\text{meas}} = B_{\text{dirty}} \ast \{B_{\text{primary}} \cdot I_{\text{source}}\} + N. \]

Information lost:

⇒ 1. Impossible to recover the intrinsic source structure!
⇒ 2. Infinite number of solutions!

Deconvolution goal: Finding a sensible intensity distribution compatible with the intrinsic source one.

Deconvolution needs:

• Some a priori assumptions about the source intensity distribution;
• As much as possible knowledge of
  – \( B_{\text{dirty}} \) (OK in radioastronomy);
  – Noise properties.

The best solution: A Gaussian \( B_{\text{dirty}} \Rightarrow \) No deconvolution needed!
Deconvolution: II. MEM principle

*a priori* assumptions: Smoothed and positive intensity.

Idea:

“Select from the images that agree with the measured visibilities to within the noise level the one that maximizes entropy.”

Algorithm:

- **Entropy:**
  \[ S = - \sum_{ij} I_{ij} \log(I_{ij}/M_{ij}) \] with \( M = \) first guess image.

- **Constraint:**
  \[ \sum_k \frac{|V(u_k,v_k) - \tilde{I}(u_k,v_k)|^2}{\sigma_k^2} = \text{number of visibilities} \]
  with \( \tilde{I} = 2D \text{ FT}(I) \).
Deconvolution: II. MEM properties

Advantages:

- Fast:
  Computational load $\propto N \ln(N)$ with $N = \text{number of pixels}$.
- Easy to generalize (Arrays with different antenna diameters).
- Flatten low-level extended emission.
- Resolve peaks.

Inconvenients:

- Angular resolution increases with peak height.
- Unable to clean ripples (e.g. point source sidelobes) in extended emission.
- Biased residuals:
  $\Rightarrow$ Noise increase and spurious emission at low signal.
- Impossibility to deal with absorption features.
- Poor performance with limited $uv$ coverage
  $\Rightarrow$ Not used at PdBI.
Deconvolution: III. The Basic CLEAN Algorithm

*a priori* assumption: Source = Collection of point sources.

Idea: “Matching pursuit”.

Algorithm:

1. Initialize
   - the residual map to the dirty map;
   - the Clean component list to an empty (NULL) value;
2. Identify pixel of $|I_{\text{max}}|$ in residual map as a point source;
3. Add $\gamma.I_{\text{max}}$ to clean component list;
4. Subtract $\gamma.I_{\text{max}}$ from residual map;
5. Go back to point 2 while stopping criterion is not matched;
6. Convolution by Clean beam (*a posteriori* regularization);
5. Addition of residual map to enable:
   - Correction when cleaning is too superficial;
   - Noise estimation.
Deconvolution: III. The Basic Clean Algorithm

1. First Illustration

![Intrinsic Sky Intensity Graph]

- Jy
- Relative Sky Position (Arcsec)

Imaging, Deconvolution & Image Analysis

J. Pety, 2010
Deconvolution: III. The Basic Clean Algorithm

1. First Illustration

![Graph showing a comparison between Dirty and Clean beams. The graph has an x-axis labeled 'Relative Sky Position (Arcsec)' ranging from -10 to 10, and a y-axis labeled 'Arbitrary Unit.' The graph displays two peaks, one labeled 'Dirty' and another labeled 'Clean (Gaussian Fit).' The graph demonstrates the deconvolution process by showing how the Dirty beam is transformed into a Clean beam through a Gaussian fit.]
Deconvolution: III. The Basic Clean Algorithm

1. First Illustration

Dirty Map

Jy

Relative Sky Position (Arcsec)
Deconvolution: III. The Basic Clean Algorithm

1. First Illustration

Residual Map

Clean Component List

Jy

Relative Sky Position (Arcsec)
Deconvolution: III. The Basic Clean Algorithm

1. First Illustration

Residual Map

Clean Component List

Maximum Localization

Jy

Relative Sky Position (Arcsec)
Deconvolution: III. The Basic Clean Algorithm

1. First Illustration

Residual Map

Clean Component List

Multiply Maximum by Dirty Beam
Deconvolution: III. The Basic Clean Algorithm

1. First Illustration

Residual Map

Clean Component List

Subtraction

Imaging, Deconvolution & Image Analysis

J. Pety, 2010
Deconvolution: III. The Basic Clean Algorithm
1. First Illustration

Residual Map

Clean Component List

Maximum Localization

Jy

Relative Sky Position (Arcsec)
Deconvolution: III. The Basic Clean Algorithm

1. First Illustration

Residual Map

Multiply Maximum by Dirty Beam

Clean Component List
Deconvolution: III. The Basic Clean Algorithm

1. First Illustration

Residual Map

Clean Component List

Subtraction

Imaging, Deconvolution & Image Analysis

J. Pety, 2010
Deconvolution: III. The Basic Clean Algorithm
1. First Illustration

Residual Map

Clean Component List

Convolution by the Clean Beam

Jy

Relative Sky Position (Arcsec)

-20  0  20

0  2  4  6

-20  0  20

0  2  4  6

Imaging, Deconvolution & Image Analysis J. Pety, 2010
Deconvolution: III. The Basic Clean Algorithm
2. Second Illustration

![Graph showing Intrinsic Sky Intensity vs. Relative Sky Position (Arcsec)]
Deconvolution: III. The Basic Clean Algorithm
2. Second Illustration

Residual Map

Clean Component List
Deconvolution: III. The Basic Clean Algorithm

2. Second Illustration

Residual Map

Clean Component List

Imaging, Deconvolution & Image Analysis  J. Pety, 2010
Deconvolution: III. The Basic Clean Algorithm

2. Second Illustration

- Residual Map
- Clean Component List

Maximum Localization

Imaging, Deconvolution & Image Analysis  J. Pety, 2010
Deconvolution: III. The Basic Clean Algorithm

2. Second Illustration

Residual Map

Clean Component List

Multiply Maximum by Dirty Beam
Deconvolution: III. The Basic Clean Algorithm

2. Second Illustration

![Residual Map and Clean Component List](image)
Deconvolution: III. The Basic Clean Algorithm
2. Second Illustration

Residual Map

Clean Component List

Maximum Localization

Imaging, Deconvolution & Image Analysis
J. Pety, 2010
Deconvolution: III. The Basic Clean Algorithm

2. Second Illustration

Residual Map

Multiply Maximum by Dirty Beam

Clean Component List

Imaging, Deconvolution & Image Analysis

J. Pety, 2010
Deconvolution: III. The Basic Clean Algorithm
2. Second Illustration

Residual Map

Clean Component List

Subtraction

Relative Sky Position (Arcsec)
Deconvolution: III. The Basic Clean Algorithm

2. Second Illustration

Residual Map

Clean Component List

Convolution by the Clean Beam

Imaging, Deconvolution & Image Analysis

J. Pety, 2010
Deconvolution: III. The Basic Clean Algorithm

3. Little Secrets

Convergence:
Too superficial cleaning ⇒ Approximate results.
Too deep cleaning ⇒ Divergence.

Difference (Cleaned−Sky)

RMS = 0.21 Jy
Deconvolution: III. The Basic Clean Algorithm
3. Little Secrets

Addition of residual map:
Improvement when convergence not reached;
Noise estimation.
Deconvolution: III. The Basic Clean Algorithm
3. Little Secrets

Addition of residual map:
Improvement when convergence not reached;
Noise estimation.

Difference (Cleaned−Sky)

RMS = 0.08 Jy

RMS = 0.08 Jy
Choice of clean beam:
Gaussian of FWHM matching the synthesized beam size.
⇒ Super resolution strongly discouraged.
Deconvolution: III. The Basic Clean Algorithm

3. Little Secrets

Negative clean components are mandatory.
Deconvolution: III. The Basic Clean Algorithm
3. Little Secrets

Negative clean components are mandatory.

Residual Map

Clean Component List

Imaging, Deconvolution & Image Analysis  J. Pety, 2010
Deconvolution: III. The Basic Clean Algorithm
3. Little Secrets

Negative clean components are mandatory.

Residual Map

Clean Component List

Multiply Maximum by Dirty Beam
Deconvolution: III. The Basic Clean Algorithm
3. Little Secrets

Negative clean components are mandatory.

Residual Map

Clean Component List
Deconvolution: III. The Basic Clean Algorithm
3. Little Secrets

Negative clean components are mandatory.
Negative clean components are mandatory.
Deconvolution: III. The Basic Clean Algorithm

3. Little Secrets

Negative clean components are mandatory.
Negative clean components are mandatory.
Deconvolution: III. The Basic Clean Algorithm
3. Little Secrets

Negative clean components are mandatory.
Deconvolution: III. The Basic Clean Algorithm
3. Little Secrets

Negative clean components are mandatory.

![Graph showing difference (cleaned-sky) with RMS = 5.E-03 Jy]
Deconvolution: III. The Basic Clean Algorithm
4. Other Little Secrets

• Stopping criterions:
  – Total number of Clean components;
  – $|I_{\text{max}}| < \text{fraction of noise (when noise limited)}$;
  – $|I_{\text{max}}| < \text{fraction of dirty map max (when dynamic limited)}$.

• Loop gain: Good results when $\gamma \sim 0.1 - 0.3$.

• Cleaned region: Only the inner quarter of the dirty image.

• Support: Definition of a region where CLEAN components are searched.
  – $A \text{ priori}$ information $\Rightarrow$ Help CLEAN convergence.
  – But bias if support excludes signal regions
    $\Rightarrow$ Be wise!
Deconvolution: III. The Basic Clean Algorithm
5. A True Example without support
Deconvolution: III. The Basic Clean Algorithm
5. A True Example without support (zoom)
Deconvolution: III. The Basic Clean Algorithm
5. A True Example with right support

![Graphs showing dirty beam, dirty image, clean components, residuals, and clean image with respective flux.]
Deconvolution: III. The Basic Clean Algorithm
5. A True Example with wrong support

![Image showing dirty beam, dirty image, clean component number, cumulated flux, clean components, residuals, and clean image.](image-url)
Deconvolution: IV. CLEAN Variants

Basic:

- **HOGBOM** (Hogböm 1974)
  Robust but slow.

Faster Search Algorithms:

- **CLARK** (Clark 1980)
  Fast but instable (when sidelobes are high).

- **MX** (Cotton & Schwab 1984)
  Better accuracy (Source removal in the $uv$ plane), but slower (gridding steps repeated).

Better Handling of Extended Sources:

- **MULTI** (Multi-Scale Clean by Cornwell 1998)
  Multi-resolution approach.
Deconvolution: IV. CLEAN Variants (continued)

Exotic use at PdBI:

- **SDI** (Steer, Dewdney, Ito 1984)
  Created to minimize stripes.

- **MRC** (Multi-Resolution Clean by Wakker & Schwarz 1988)
  Too simple multi-resolution approach.
Deconvolution: V. Recommended Practices

- **Method**: Start with CLARK and turn to HOGBOM in case of high side-lobes.

- **Support**:
  - Start without one.
  - Define one on your first clean image if really needed (*i.e.* difficulties of convergence).

- **Stopping criterion**:
  - Use a large enough number of iterations to ensure convergence.
  - Clean down to the noise level unless a very strong source is present.

- **Misc**: Consult an expert until you become one.
### Visualization and Image Analysis

**Fourier Transform and Deconvolution:**
The two key issues in imaging.

<table>
<thead>
<tr>
<th>Stage</th>
<th>Implementation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calibrated Visibilities</td>
<td></td>
</tr>
<tr>
<td>↓ Fourier Transform</td>
<td><strong>GO UVSTAT, GO UVMAP</strong></td>
</tr>
<tr>
<td>Dirty beam &amp; image</td>
<td></td>
</tr>
<tr>
<td>↓ Deconvolution</td>
<td><strong>GO CLEAN</strong></td>
</tr>
<tr>
<td>Clean beam &amp; image</td>
<td></td>
</tr>
<tr>
<td>↓ Visualization</td>
<td><strong>GO BIT, GO VIEW</strong></td>
</tr>
<tr>
<td>↓ Image analysis</td>
<td><strong>GO NOISE, GO FLUX, GO MOMENTS</strong></td>
</tr>
<tr>
<td>Physical information on your source</td>
<td></td>
</tr>
</tbody>
</table>
Photometry: I Generalities

- Brightness = Intensity (e.g. Power = $I_\nu(\alpha, \beta)dAd\Omega d\nu$)

- Flux unit: 1 Jy = $10^{-26}$ W m$^{-2}$ Hz$^{-1}$.

- Source flux measured by a single–dish antenna:
  \[ F_\nu = B \ast I_\nu \] with $B$ the antenna beam.

- Relationship between measured flux and temperature scales:
  \[ T_A = \frac{\lambda^2}{2k\Omega_A} F_\nu, \quad T_A^* = \frac{\lambda^2}{2k\Omega_{2\pi}} F_\nu \quad \text{and} \quad T_{mb} = \frac{\lambda^2}{2k\Omega_{mb}} F_\nu \] because
  - $P_\nu = \frac{1}{2} A_e F_\nu$ Power detected by the single–dish antenna.
  - $P'_\nu = kT$ Power emitted by a resistor at temperature $T$.
  - $P_\nu = P'_\nu \Rightarrow T_A = \frac{A_e}{2k} F_\nu$.
  - $\chi^2 = A_e \Omega_A$ (diffraction).
  - $\Omega_{2\pi} = F_{\text{eff}} \Omega_A$ or $F_{\text{eff}} = \frac{\text{Forward beam}}{\text{Total beam}}$.
  - $\Omega_{mb} = B_{\text{eff}} \Omega_A$ or $B_{\text{eff}} = \frac{\text{Main beam}}{\text{Total beam}}$. 
Visibility unit: $\text{Jy}$ because:

$$V = 2\text{D FT}\left\{ B_{\text{primary}}.I_{\text{source}} \right\}$$
$$= \int \int B_{\text{primary}}(\sigma).I_{\text{source}}(\sigma) \exp(-i2\pi b.\sigma/c) d\Omega.$$

Effect of flux calibration errors on your image:

- Multiplicative factor if uniform in $uv$ plane.
- Convolution (i.e. distorsion) else.
Ill–defined because:

- $S(u = 0, v = 0) = 0 \Rightarrow$ Area of the dirty beam is 0!
- $V(u = 0, v = 0) = 0 \Rightarrow$ Total flux of the dirty image is 0!
  \[ \Rightarrow \text{A source of constant intensity will be fully filtered out.} \]
- A single point source of 1 Jy appears with peak intensity of 1.
- Several close-by point sources of 1 Jy appears with peak intensities different of 1.
Photometry: IV Clean map

*(my dream: Don’t take it seriously)*

\[ I_{\text{clean}} = \frac{1}{\Omega_{\text{clean}}} (B_{\text{clean}} \ast I_{\text{point}}) \]: *i.e.* convolution of a set of point sources (mimicking the sky intensity distribution) by the clean beam.

Behavior: Brightness, *i.e.* Source flux measured in a given solid angle (*i.e.* 1 steradian).

Unit: Jy/sr

Consequences:

- Source flux computation by integration inside a support:

  \[ \text{Flux} = \sum_{ij \in S} I_{\text{clean}} \, d\Omega \]

  \[ \text{[Jy]} \quad \text{[Jy/sr]} \quad \text{[sr]} \]

  with \( d\Omega \) the image pixel surface.

- From Brightness to temperature: \( T_{\text{clean}} = \frac{\lambda^2}{2k} I_{\text{clean}} \)
Photometry: IV Clean map (reality)

\[ I_{\text{clean}} = B_{\text{clean}} \ast I_{\text{point}}: \text{i.e. convolution of a set of point sources (mimicking the sky intensity distribution) by the clean beam.} \]

Behavior: Brightness, \text{i.e. Source flux measured in a given solid angle (\text{i.e. clean beam}).}

Unit: Jy/beam with 1 beam = \( \Omega_{\text{clean}} \) sr.

Consequences:

- Source flux computation by integration inside a support:

\[
\text{Flux} = \sum_{ij \in S} I_{\text{clean}} \cdot \frac{d\Omega}{\Omega_{\text{clean}}} \text{ [Jy] [Jy/beam] [beam]}
\]

with \( \frac{d\Omega}{\Omega_{\text{clean}}} \) the nb of beams in the surface of an image pixel.

- From Brightness to temperature: \( T_{\text{clean}} = \frac{\lambda^2}{2k\Omega_{\text{clean}}} I_{\text{clean}} \)
Photometry: IV Clean map

Consequences of a Gaussian clean beam shape:

- No error beams, no secondary beams.
- $T_{\text{clean}}$ is a main beam temperature.

Natural choice of clean beam size: Synthesized beam size
(i.e. fit of the central peak of the dirty beam).
⇒ Minimize unit problems when adding the dirty map residuals.

Caveats of flux measurements:

- CLEAN does not conserve flux
  (i.e. CLEAN extrapolates unmeasured short spacings).
- Large scales are filtered out (source size > 1/3 primary beam size ⇒ need of short spacings, cf. lecture by F. Gueth).
- $I_{\text{clean}} = B_{\text{primary}} \cdot I_{\text{source}} + N$
  ⇒ Primary beam correction may be needed:
  $I_{\text{clean}} / B_{\text{primary}} = I_{\text{source}} + N / B_{\text{primary}}$ ⇒ Varying noise!
- Seeing scatters flux.
Photometry: V Importance of Extended, Low Level Intensity
\[ \delta T = \frac{\lambda^2 \sigma}{2k \Omega} \quad \text{with} \quad \sigma = \frac{2k}{\eta} \frac{T_{\text{sys}}}{\sqrt{\Delta t \Delta \nu} \sqrt{N_{\text{ant}} (N_{\text{ant}} - 1) A}} \]

\( \delta T \)  Brightness noise [K].
\( \lambda \)  Wavelength.
\( k \)  Boltzmann constant.
\( \Omega \)  Synthesized beam solid angle.
\( A \)  Antenna area.

and \( \eta \)  Global efficiency (\( = \) Quantum \( \times \) Antenna \( \times \) Atm. Decorrelation).

\( \sigma \)  Flux noise [Jy].
\( T_{\text{sys}} \)  System temperature.
\( \Delta t \)  On-source integration time.
\( \Delta \nu \)  Channel bandwidth.
\( N_{\text{ant}} \)  Number of antennas.
Noise: \( \Delta T \) to compare instruments

\[
\delta T = \frac{\lambda^2 \sigma}{2k\Omega} \quad \text{with} \quad \sigma = \frac{2k}{\eta} \frac{T_{sys}}{\sqrt{\Delta t \Delta \nu} \sqrt{N_{ant}(N_{ant} - 1)A}}
\]

Wavelength: 1 mm. \( T_{sys} = 150 \) K. Decorrelation = 0.8.

<table>
<thead>
<tr>
<th>Instrument</th>
<th>Bandwidth</th>
<th>( \sigma )</th>
<th>On-source time</th>
</tr>
</thead>
<tbody>
<tr>
<td>PdBI 2009</td>
<td>8 GHz</td>
<td>1.0 mJy/Beam</td>
<td>3 min</td>
</tr>
<tr>
<td>ALMA 2012</td>
<td>16 GHz</td>
<td>1.0 mJy/Beam</td>
<td>3 sec</td>
</tr>
<tr>
<td>ALMA 2012</td>
<td>16 GHz</td>
<td>0.12 mJy/Beam</td>
<td>3 min</td>
</tr>
</tbody>
</table>

One order of magnitude (\( \sim 8\times \)) sensitivity increase in continuum.
Noise: III. $\delta T$ to prepare observations: 1. Continuum

$$\delta T = \frac{\lambda^2 \sigma}{2k\Omega}$$

with

$$\sigma = \frac{2k}{\eta \sqrt{\Delta t \Delta \nu} \sqrt{N_{\text{ant}}(N_{\text{ant}} - 1) A}}$$

Wavelength: 1 mm. $T_{\text{sys}} = 150$ K. Decorrelation = 0.8.

<table>
<thead>
<tr>
<th>Instrument</th>
<th>Bandwidth</th>
<th>Resol.</th>
<th>$\delta T$</th>
<th>On time</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>PdBI 2009</td>
<td>8 GHz</td>
<td>0.30&quot;</td>
<td>30 mK</td>
<td>3 hrs</td>
<td></td>
</tr>
<tr>
<td>ALMA 2012</td>
<td>16 GHz</td>
<td>0.30&quot;</td>
<td>30 mK</td>
<td>3 min</td>
<td>Low contrast, many objects</td>
</tr>
<tr>
<td>ALMA 2012</td>
<td>16 GHz</td>
<td>0.30&quot;</td>
<td>4 mK</td>
<td>3 hrs</td>
<td>High contrast, same object</td>
</tr>
<tr>
<td>ALMA 2012</td>
<td>16 GHz</td>
<td>0.03&quot;</td>
<td>30 mK</td>
<td>500 hrs</td>
<td>5.7% of a civil year</td>
</tr>
<tr>
<td>ALMA 2012</td>
<td>16 GHz</td>
<td>0.03&quot;</td>
<td>400 mK</td>
<td>3 hrs</td>
<td>Intermediate sensitivity</td>
</tr>
<tr>
<td>ALMA 2012</td>
<td>16 GHz</td>
<td>0.10&quot;</td>
<td>30 mK</td>
<td>3 hrs</td>
<td>Intermediate resolution</td>
</tr>
</tbody>
</table>

Almost one order of magnitude ($\sim 8 \times$) sensitivity increase

$\Rightarrow$ A factor $\sim 3$ resolution increase

(same integration time, same noise level).

Wolf et al. 2002, 0.02" in 3 hrs.
Noise: **III.** $\delta T$ to prepare observations: **2. Line**

\[
\delta T = \frac{\lambda^2 \sigma}{2k \Omega} \quad \text{with} \quad \sigma = \frac{2k T_{\text{sys}}}{\eta \sqrt{\Delta t \Delta \nu} \sqrt{N_{\text{ant}}(N_{\text{ant}} - 1) A}}
\]

Channel width: 0.8 km s$^{-1}$. Wavelength: 1 mm. Decorrelation = 0.8.

<table>
<thead>
<tr>
<th>Instrument</th>
<th>Resolution</th>
<th>$\delta T$</th>
<th>On-source time</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>PdBI now</td>
<td>1''</td>
<td>0.3 K</td>
<td>2 hrs</td>
<td></td>
</tr>
<tr>
<td>ALMA 2012</td>
<td>1''</td>
<td>0.3 K</td>
<td>3.5 min</td>
<td>Same line, many objects</td>
</tr>
<tr>
<td>ALMA 2012</td>
<td>1''</td>
<td>0.05 K</td>
<td>2 hrs</td>
<td>Fainter lines, same object</td>
</tr>
<tr>
<td>ALMA 2012</td>
<td>0.1''</td>
<td>0.3 K</td>
<td>575 hrs</td>
<td>6.5% of a civil year!</td>
</tr>
<tr>
<td>ALMA 2012</td>
<td>0.1''</td>
<td>5 K</td>
<td>2 hrs</td>
<td>Intermediate sensitivity</td>
</tr>
<tr>
<td>ALMA 2012</td>
<td>0.4''</td>
<td>0.3 K</td>
<td>2 hrs</td>
<td>Intermediate resolution</td>
</tr>
</tbody>
</table>

A factor $\sim 6$ sensitivity increase

⇒ A factor $\sim 2.4$ resolution increase

(same integration time, same noise level).
**Noise: IV. Advices**

\[
\delta T = \frac{\lambda^2 \sigma}{2k \Omega} \quad \text{with} \quad \sigma = \frac{2k}{\eta \sqrt{\Delta t \Delta \nu}} \sqrt{\frac{T_{\text{sys}}}{N_{\text{ant}}(N_{\text{ant}} - 1) A}}
\]

- For your estimation:
  - Use a sensitivity estimator!
  - The estimator is probably optimistic!
  - Use \( \delta T \) not \( \sigma \).
Writing the Paper: Your job!
Mathematical Properties of Fourier Transform

1 Fourier Transform of a product of two functions
   = convolution of the Fourier Transform of the functions:
     If \( F_1 \xrightarrow{\text{FT}} \tilde{F}_1 \) and \( F_2 \xrightarrow{\text{FT}} \tilde{F}_2 \), then \( F_1 \cdot F_2 \xrightarrow{\text{FT}} \tilde{F}_1 \ast \tilde{F}_2 \).

2 Sampling size \( \xrightarrow{\text{FT}} \) Image size.

3 Bandwidth size \( \xrightarrow{\text{FT}} \) Pixel size.

4 Finite support \( \xrightarrow{\text{FT}} \) Infinite support.

5 Fourier transform evaluated at zero spatial frequency
   = Integral of your function.
     \[ V(u = 0, v = 0) \xrightarrow{\text{FT}} \sum_{ij \in \text{image}} I_{ij}. \]
Photographic Credits and References

- J. D. Kraus, “Radio Astronomy”.