Single-dish antenna at (sub)mm wavelengths

P. Hily-Blant
Institut de Planétologie et d’Astrophysique de Grenoble
Université Joseph Fourier

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Introduction
A single-dish antenna
Spectral surveys

Caux et al 2011 (IRAM spectral survey of l16293)
Spectro-imaging
Few fW (1e-15 W)

Gain ~ 120 dB
Few mW

Receiver cabin

Backends

Frontends
Wishes

- Measure some power emitted in a (narrow) frequency range from a particular location
- Possibly want to make some (spectral/continuum) maps
- Eventually determine some chemical and/or physical properties

Questions

- Measurement fidelity ?
- Calibration (amplitude, frequency)
- Spatial resolution ?
General properties
Primary vs Secondary focus

*Primary focus*

**Ground**

\[ T_g = 300\text{K} \]

*Secondary focus*

**Sky**

\[ T_s = \tau \]

\[ T_{atm} = 30-100\text{K} \]
(dis)Advantages

- $f/D \rightarrow f_e/D = m \times f/D$
  - IRAM-30m, $m = 27.8$
  - $f/D = 0.35$, $f_e/D \approx 10$ or 300 m
- Rx alignment easier: $1''$ on the sky $\leftrightarrow f_e/206265$ mm in focal plane
- Increase effective area (or on-axis gain)
- Decrease spillover
- *but* increase mechanical load
- Obstruction by subreflector ($\varnothing = 2$ m at 30-m) $\Rightarrow$ wider main-beam
World map of radiotelescopes
Large aperture: $f/D \lesssim 1$

<table>
<thead>
<tr>
<th>Obs.</th>
<th>$D$ (m)</th>
<th>$\nu$ (GHz)</th>
<th>$\lambda$ (mm)</th>
<th>HPBW (″)</th>
<th>Latitude (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>IRAM</td>
<td>30</td>
<td>70 – 345</td>
<td>4 – 0.7</td>
<td>35 – 7</td>
<td>+37</td>
</tr>
<tr>
<td>APEX</td>
<td>12</td>
<td>230 – 1200</td>
<td>1.3 – 0.3</td>
<td>30 – 6</td>
<td>−22</td>
</tr>
<tr>
<td>JCMT$^\dagger$</td>
<td>15</td>
<td>210 – 710</td>
<td>2 – 0.2</td>
<td>20 – 8</td>
<td>+20</td>
</tr>
<tr>
<td>CSO$^\dagger$</td>
<td>10.4</td>
<td>230 – 810</td>
<td>1.3 – 0.4</td>
<td>30 – 10</td>
<td>+20</td>
</tr>
<tr>
<td>Herschel$^\dagger$</td>
<td>3.5</td>
<td>500–2000</td>
<td>0.6 – 0.1</td>
<td>43 – 11</td>
<td>space</td>
</tr>
</tbody>
</table>
Terminology (1): receivers

- Central frequency
  \[ \nu_0 = 80 - 2000 \text{ GHz} \]
- Instantaneous bandwidth:
  \[ \Delta \nu = 1 - 32 \text{ GHz} \]
- Bolometers
  \[ \Delta \nu \approx 50 \text{ GHz} \]
- One polarization (linear, circular)
- Taper (apodization at the rim)
Terminology (2): backends

- Spectrometers:
  - digital: autocorrelators (AC), Fast Fourier Transform Spectrometer (FTS)
  - (analogical: filter banks (FB), acousto-optic (AOS))
- Spectral resolution
  \[ \delta \nu \approx 3 - 2000 \text{ kHz} \]
- Large resolution power
  \[ R = \nu_0 / \delta \nu \approx 10^5 - 10^8 \]
Collected power from a source

Consider an unpolarized point source

- Monochromatic (and monomode) power collected by an area $A_e$:
  \[
  p_\nu = \frac{1}{2} A_e \cdot S_\nu \quad [\text{W Hz}^{-1}]
  \]

- Flux density $S_\nu$ measured in Jy:
  \[
  1 \text{Jy} = 10^{-26} \text{J s}^{-1} \text{m}^{-2} \text{Hz}^{-1}
  \]

- Power in the bandwidth $\Delta \nu$:
  \[
  p = \frac{1}{2} A_e \cdot S_\nu \cdot \Delta \nu \quad [\text{W}]
  \]

- Note: A 1 Jy source observed with a $10^3 \text{ m}^{-2}$ radiotelescope during 40 yrs (e.g. Orion) with 50 MHz bandwidth $\Rightarrow \approx 40 \text{ GeV}$
Consider an unpolarized extended source

- Monochromatic (and monomode) power collected by an area $A_e$ from solid angle $\delta\Omega$:
  \[ \delta p_\nu = \frac{1}{2} A_e \cdot l_\nu \cdot \delta\Omega \quad [\text{W Hz}^{-1}] \]

- Brightness $l_\nu$ measured in Jy sr$^{-1}$:
  \[ 1 \text{ Jy sr}^{-1} = 10^{-26} \text{ J s}^{-1} \text{ m}^{-2} \text{ Hz}^{-1} \text{ sr}^{-1} \]
Collected power from a source

Effective area of the antenna: \( A_e = \eta A_{\text{geom}} \)

\[ \eta < 1 \]

Question: \( \eta = ? \)
Perfect antenna
Diffraction theory (Huygens-Fresnel, Fraunhoffer approx.):\[
E_{\text{f} \rightarrow \text{f}}(l, m) \propto \mathcal{F}[E_{\text{ant}}(x, y)]
\]

$E_{\text{ant}}(x, y)$ (grading): bounded on a finite domain $\Delta r$
$
\Rightarrow E_{\text{f} \rightarrow \text{f}}(l, m) \text{ concentrated on a finite domain } \Delta \Omega$
$(\Delta r \cdot \Delta \Omega \sim 1)$

- sharp cut of the antenna domain $\Rightarrow$ oscillations (side-lobes)
- apodization or taper: decrease the level of the sidelobes, to the cost of increasing $\Delta \Omega$
Antenna power pattern

- Reciprocity: antenna in emission
- Distribution of electric field on the dish: $E_{\text{ant}}(x, y)$
- Far-field radiated by the dish: $E_{\text{f-f}}(l, m) \propto \mathcal{F}[E_{\text{ant}}(x, y)]$
- Power emitted is a function of direction: $\propto |E_{\text{f-f}}(l, m)|^2$

- Power pattern: $P(l, m) \propto |E_{\text{f-f}}(l, m)|^2$
- Beam solid angle: $\Omega_A = \int_{4\pi} P(\Omega) \, d\Omega \leq 4\pi$
- Effective area: $A(l, m) = A_e \cdot P(l, m) \leq A_e$
- Fundamental relation:

$$A_e \Omega_A = \lambda^2$$
Power pattern $\mathcal{P}(l, m)$
Power collected by an antenna (2)

Given a source of brightness $I_\nu(l, m) = I_\nu(\Omega)$

- Source flux density:
  $S_\nu = \int_{\Omega_s} I_\nu(\Omega) \, d\Omega$

- Observed flux density:
  $S_{\text{obs}} = \int_{\Omega_s} \mathcal{P}(\Omega) \, I_\nu(\Omega) \, d\Omega < S_\nu$

- Power received from $d\Omega_i$:
  $d\rho_\nu = \frac{1}{2} A(\Omega_i) \, I_\nu(\Omega_i) \, d\Omega_i$

- Incoherent emission: add intensities

Pointing towards a fixed position of the source at a fixed position $l = 0, m = 0$

$\rho_\nu(l = 0, m = 0) = \frac{A_e}{2} \int_{\Omega_s} \mathcal{P}(\Omega) \, I_\nu(\Omega) \, d\Omega = \frac{1}{2} A_e S_{\text{obs}}$
- Antenna tilted towards $\Omega_0 = (l_0, m_0)$
- Power received from the direction $\Omega_i$

$$dp_{\nu}(\Omega_i) = \frac{1}{2} A(\Omega_0 - \Omega_i) I_{\nu}(\Omega_i) \ d\Omega_i$$

- Incoherent emission: add intensities

Scanning a source leads to a convolution

$$S_{\text{obs}}(\Omega_0) = \int_{\Omega_S} P(\Omega - \Omega_0) I_{\nu}(\Omega) \ d\Omega$$

$$p_{\nu}(\Omega_0) = \frac{A_e}{2} \int_{\Omega_S} P(\Omega_0 - \Omega) I_{\nu}(\Omega) \ d\Omega = \frac{1}{2} A_e S_{\text{obs}}(\Omega_0)$$
Note: When quoting sizes from observations, must quote the deconvolved size (when needed).
Real antenna
Systematic deformations (1)

Defocus

(a) [Graph showing variation with angle (θ in arcsec)]

(b) [Graph showing loss main beam power versus ΔF (in λ)]
Coma: misaligned subref.
Systematic deformations (3)

Astigmatism

⇒ Beam deformation (no pointing error)
Beam pattern

- Main lobe
- Secondary lobes (finite surface antenna)
- Error lobes (surface irregularities)
- main-beam collects **less** power
- if correlation length $\ell$
  $\Rightarrow$ Gaussian error-beam
  $\Theta_{EB} \approx \lambda/\ell$

**real beam = main-beam + error-beam(s)**

- Questions:
  - Power collected in each e-beam ?
  - FWHMs of the e-beams ?

Greve et al 1998
Error-Beams at IRAM-30m

1.3mm

\[ \theta_\text{in} (\degr) \]

Normalized integrated power \( P(\theta) \)

\[ \log(\theta_s / \theta_D) \]

- 40% in MB
- 30% into 3rd EB
Temperature scales
CARBON MONOXIDE IN THE ORION NEBULA

R. W. Wilson, K. B. Jefferts, and A. A. Penzias
Bell Telephone Laboratories, Inc., Holmdel, New Jersey, and
Crawford Hill Laboratory, Murray Hill, New Jersey

Received 1970 June 5

ABSTRACT

We have found intense 2.6-mm line radiation from nine galactic sources which we attribute to carbon monoxide.

Fig. 1.—Spectrum of CO radiation in the Orion Nebula made with the NRAO forty-channel line receiver. The center frequency is 115, 267.2 MHz.

Fig. 2.—Distribution in right ascension of the peak antenna temperature of CO radiation at a declination of $-5^\circ 24' 21''$. 
Antenna temperature: $T_A$

- Johnson noise in terms of an equivalent temperature: the average power transferred from a conductor (in thermal equilibrium) to a line within $\delta\nu$: $\delta p = kT \delta\nu$
- Antenna temperature defined by $p_\nu = kT_A \quad [W \cdot Hz^{-1}] = [J] = [J \cdot K^{-1}] [K]$
- On the other hand: $p_\nu = \frac{A_e}{2} (\mathcal{P} \ast I_\nu) = \frac{\chi^2}{2\Omega_A} (\mathcal{P} \ast I_\nu)$
- Antenna temperature:
  \[
  T_A(\Omega_0) = \frac{A_e}{2k} \int_{\Omega_S} I_\nu(\Omega) \mathcal{P}(\Omega - \Omega_0) \, d\Omega
  \]
- Using $A_e\Omega_A = \chi^2$, we may write:
  \[
  T_A(\Omega_0) = \frac{1}{\Omega_A} \int_{\Omega_S} \frac{\chi^2}{2k} I_\nu(\Omega) \mathcal{P}(\Omega - \Omega_0) \, d\Omega
  \]
Speaking in terms of temperatures

Black-body radiation at temperature $T$: $B_\nu(T) = \frac{2\hbar \nu^3}{c^2} \frac{1}{e^{h\nu/kT} - 1}$

- Dimensions of brightness: $\text{J s}^{-1} \text{m}^{-2} \text{Hz}^{-1} \text{sr}^{-1}$
- Rayleigh-Jeans approximation: $h\nu \ll kT$

$$\frac{\chi^2}{2k} B_\nu(T) \approx T$$

- Flux density of a black-body:

$$S_\nu = \int_{\Omega_S} B_\nu(T, \Omega) \, d\Omega = 4\pi B_\nu(T)$$
Definitions: $T_B$, $T_R$

- **Brightness temperature** $T_B$ of source brightness $I_\nu$:

  \[
  I_\nu(\Omega) = B_\nu(T_B)
  \]

- **Radiation temperature**, $T_R$, in the Rayleigh-Jeans regime approximation:

  \[
  I_\nu(\Omega) = \frac{2k\nu^2}{c^2} T_R(\Omega) = \frac{2k}{\lambda^2} T_R
  \]

  \[
  [\text{J s}^{-1} \text{ m}^{-2} \text{ Hz}^{-1} \text{ sr}^{-1}]
  \]

- Relationship between $T_B$ and $T_R$:

  \[
  T_R = J_\nu(T_B) = \frac{h\nu}{k} \frac{1}{\exp(h\nu/kT_B) - 1} = \frac{T_0}{\exp(T_0/T_B) - 1}
  \]

  In the following: $I_\nu(\Omega) \rightarrow T_R(\Omega)$
R-J approximation in the (sub)mm

\[ \frac{2k}{\lambda^2} J_\nu(T_B) = B_\nu(T_B) \]

\[ R-J \text{ approximation in the (sub)mm} \]
Consequences

- Monochromatic power received by the antenna:
  \[ p_\nu(\Omega_0) = \frac{k}{\Omega_A} \int_{\Omega_S} \mathcal{P}(\Omega) T_R(\Omega_0 - \Omega) \, d\Omega \]

- Observed flux density \((p_\nu = 1/2A_e S_\nu)\)
  \[ S_{\text{obs}}(\Omega_0) = \frac{2k}{\lambda^2} \int_{\Omega_S} \mathcal{P}(\Omega) T_R(\Omega_0 - \Omega) \, d\Omega \]
Antenna temperature: $T_A$

Antenna temperature:

$$ T_A(\Omega_0) = \frac{A_e}{\lambda^2} \int_{\Omega_S} \mathcal{P}(\Omega) T_R(\Omega - \Omega_0) \, d\Omega $$

Using $A_e \Omega_A = \lambda^2$, we may write:

$$ T_A(\Omega_0) = \frac{1}{\Omega_A} \int_{\Omega_S} \mathcal{P}(\Omega) T_R(\Omega - \Omega_0) \, d\Omega $$

Note that:

$$ T_A(\Omega_0) = \frac{\int_{\Omega_S} \mathcal{P}(\Omega) T_R(\Omega - \Omega_0) \, d\Omega}{\int_{4\pi} \mathcal{P}(\Omega) \, d\Omega} $$
From $T_A$ to $T'_A$

**At frequency $\nu$:**

$$T_A = \eta_s \left\{ T_R e^{-\tau_\nu} + (1 - e^{-\tau_\nu}) T_{\text{atm}} \right\} + (1 - \eta_s) T_{\text{gr}}$$

**Correct for atmospheric attenuation:**

$$T'_A = T_A e^{\tau_\nu}$$

**Note:** for space-based telescopes (e.g. HIFI/Herschel): $T'_A = T_A$
Correct for rear-sidelobes: measure the monochromatic power received from the forward $2\pi$ sr. Hence, \( \Omega_A = {\mathcal P}_{4\pi} \rightarrow {\mathcal P}_{2\pi} = \int_{2\pi} {\mathcal P}(\Omega) \, d\Omega \):

\[
T_A^*(\Omega_0) = \frac{T_A'}{F_{\text{eff}}} = \frac{1}{{\mathcal P}_{2\pi}} \int_{\Omega_S} {\mathcal P}(\Omega) \, T_R(\Omega_0 - \Omega) \, d\Omega
\]

\[
T_A^* = e^{-\tau_\nu} \frac{{\mathcal P}_{2\pi}}{{\mathcal P}_{4\pi}} T_A
\]

Forward efficiency: \( F_{\text{eff}} = {\mathcal P}_{2\pi} / {\mathcal P}_{4\pi} \)
From $T'_A$ to $T_{mb}$

- Take into account main-beam and error-lobes
- Same as $T^*_A$ but in $\Omega_{mb}$ instead of $2\pi$. Hence,
  $$\Omega_A \rightarrow \mathcal{P}_{mb} = \int_{\Omega_{mb}} \mathcal{P}(\Omega) \, d\Omega :$$
  $$T_{mb}(\Omega_0) = \frac{T'_A}{B_{eff}} = \frac{\int_{\Omega_S} \mathcal{P}(\Omega) \, T_R(\Omega_0 - \Omega) \, d\Omega}{\mathcal{P}_{mb}},$$

- Beam efficiency: $B_{eff} = \mathcal{P}_{mb}/\mathcal{P}_{4\pi}$
- Useful relation for a Gaussian beam:
  $$\Omega_{mb} = \int_{\Omega_{mb}} \mathcal{P}(\Omega) \, d\Omega = 1.133 \, \theta^2_{mb}$$
Temperature scales

Definitions

Forward efficiency: \[ F_{\text{eff}} = \frac{P_{2\pi}}{P_{4\pi}} \]

Beam efficiency: \[ B_{\text{eff}} = \frac{P_{mb}}{P_{4\pi}} \]

Consequences

\[ T_{mb} = \frac{F_{\text{eff}}}{B_{\text{eff}}} \quad T_{A}^{*} = \frac{P_{2\pi}}{P_{mb}} \quad T_{A}^{*} \]

What you measure is \( T_{A}^{*} \) or \( T_{mb} \) (usually \( \neq T_{R} \))
Limiting cases

- **Small sources**: $\Omega_S \ll \Omega_{mb}$
  \[
  T_{mb} \approx \frac{P(0)\Omega_S T_R}{P_{mb}} = T_R \frac{\Omega_S}{P_{mb}}
  \]
  Gaussian sources & beam:
  \[
  \int_{\Omega_S} P(\Omega) T_R(\Omega_0 - \Omega) \, d\Omega = 1.133(\theta^2_{sou} + \theta^2_{mb}) \quad \text{and}
  \]
  \[
  P_{mb} = 1.133\theta^2_{mb} \quad \text{hence} \quad T_R = T_{mb} \frac{\theta^2_{mb}}{\theta^2_{sou} + \theta^2_{mb}}: \text{beam dilution}
  \]

- **Large sources**: $\Omega_S \gg P_{mb}$
  \[
  T^*_A \approx T_R \frac{\int_{2\pi} P(\Omega) \, d\Omega}{P_{2\pi}} \approx T_R
  \]

- **Special case**: $\Omega_S = P_{mb}$
  \[
  T_{mb} = T_R \frac{\int_{\Omega_S} P(\Omega) \, d\Omega}{P_{mb}} = T_R
  \]
  Main-beam temperature gives the source brightness

- **General (worse) case**: $\Omega_S \sim P_{mb}$
  \[
  T^*_A = \frac{T_R}{P_{2\pi}} \int_{\Omega_S} P(\Omega) \, d\Omega
  \]
  Main-beam temperature usually quoted
  If source of uniform brightness and beam pattern known, feasible, but in real life... Which scale to use: $T^*_A$, $T_{mb}$?
### Which temperature scale?

<table>
<thead>
<tr>
<th>Source size</th>
<th>Temperature scales</th>
</tr>
</thead>
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<tr>
<td>$\Omega_S = 2\pi$</td>
<td>$T_R = T_A^*$</td>
</tr>
<tr>
<td>$\Omega_S = \Omega_{mb}$</td>
<td>$T_R = T_{mb}$</td>
</tr>
<tr>
<td>$2\pi &lt; \Omega_S$</td>
<td>$T_A^* &lt; T_R &lt; T_{mb}$</td>
</tr>
<tr>
<td>$\Omega_{mb} &lt; \Omega_S &lt; 2\pi$</td>
<td>$T_{mb} &lt; T_R$</td>
</tr>
<tr>
<td>$\Omega_{mb} &gt; \Omega_S$</td>
<td>$\Omega_{mb} &gt; \Omega_S$</td>
</tr>
</tbody>
</table>

![Graph showing temperature scales and source size relationships](image-url)
Calibration
Goals of the calibration

Overview

- Atmospheric calibration:
  - atmospheric emission/abs at frequency $\nu$: radiative transfer
  - turbulence affects the intensity through phase shifts
- Full detection chain calibration (antenna, receiver, )
  - Antenna-sky coupling: $F_{\text{eff}}$
  - Receiver: gain, noise, stability
  - Cables, backends (e.g. dark currents)

Goals

- Input is an e-m field and output at backends are counts
- Question 1: how to convert from counts to power in physical units?
- Question 2: how to correct for the atmospheric contribution?
Telescope pointing at a source receives

\[ C_{\text{sou}} = \chi \left\{ T_{\text{rec}} + F_{\text{eff}} e^{-\tau_{\nu}} T_{\text{sou}} + T_{\text{sky}} \right\} \]

where

\[ T_{\text{sky}} = F_{\text{eff}} (1 - e^{-\tau_{\nu}}) T_{\text{atm}} + (1 - F_{\text{eff}}) T_{\text{gr}} \]

- \( T_{\text{rec}} \): noise contribution from the receiver
- \( F_{\text{eff}} e^{-\tau_{\nu}} T_{\text{sou}} \): signal from the scientific target after propagation through the atmosphere
- \( T_{\text{sky}} \): signal emitted by the atmosphere \((T_{\text{atm}})\) and the ground \((T_{\text{gr}})\)

Orders of magnitude (IRAM-30m):
- \( T_{\text{atm}} \approx T_{\text{gr}} \approx 290 \text{ K} \)
- \( T_{\text{rec}} \approx 50 – 70 \text{ K at } 100 – 350 \text{ GHz at the IRAM-30m} \)
- \( \nu_{\text{GHz}} = 100 – 350: F_{\text{eff}}(\nu) \approx 90 – 80\%, B_{\text{eff}}(\nu) \approx 80 – 35\% \)
- \( T_{\text{sky}} \approx 30 – 100 \text{ K at } 100 – 230 \text{ GHz} \)
- **Note:** if \( \tau_{\nu} \ll 1 \) (good weather), \( T_{\text{sky}} \approx F_{\text{eff}} \tau_{\nu} T_{\text{atm}} \)
The “Chopper Wheel” method

Perform 3 measurements: hot, empty sky, source

\[
C_{\text{sou}} = \chi \left\{ T_{\text{rec}} + T_{\text{sky}} + F_{\text{eff}} e^{-\tau \nu} T_{\text{sou}} \right\} \\
C_{\text{atm}} = \chi \left\{ T_{\text{rec}} + T_{\text{sky}} \right\} \\
C_{\text{hot}} = \chi \left\{ T_{\text{rec}} + T_{\text{hot}} \right\}
\]

Making differences:

\[
\Delta C_{\text{sig}} = C_{\text{sou}} - C_{\text{atm}} = \chi F_{\text{eff}} e^{-\tau \nu} T_{\text{sou}} \\
\Delta C_{\text{cal}} = C_{\text{hot}} - C_{\text{atm}} = \chi (T_{\text{hot}} - T_{\text{sky}})
\]

\[
T_{\text{sou}} = T_{\text{cal}} \frac{\Delta C_{\text{sig}}}{\Delta C_{\text{cal}}}
\]

Definition of \( T_{\text{cal}} \) (correcting for atm contrib. and spillover)

\[
T_{\text{cal}} = (T_{\text{hot}} - T_{\text{sky}}) \frac{e^{\tau \nu}}{F_{\text{eff}}}
\]
**Calibration outputs (1):** $T_{\text{cal}}$

Rewrite $T_{\text{sky}}$ and $\Delta C_{\text{cal}}$:

\[
T_{\text{sky}} = T_{\text{gr}} + F_{\text{eff}} (T_{\text{atm}} - T_{\text{gr}}) - F_{\text{eff}} e^{-\tau_\nu} T_{\text{atm}}
\]

\[
T_{\text{cal}} = T_{\text{gr}} + e^{\tau_\nu} [T_{\text{gr}} - T_{\text{atm}}] + e^{\tau_\nu} / F_{\text{eff}} [T_{\text{hot}} - T_{\text{gr}}]
\]

\[
\Delta C_{\text{cal}} = \chi \left\{ (T_{\text{hot}} - T_{\text{gr}}) + F_{\text{eff}} (T_{\text{gr}} - T_{\text{atm}}) + F_{\text{eff}} e^{-\tau_\nu} T_{\text{atm}} \right\}
\]

- Assume $T_{\text{hot}} = T_{\text{atm}} = T_{\text{gr}}$.
- Then $T_{\text{cal}} = T_{\text{hot}}$

\[
T_{\text{sou}} = T_{\text{hot}} \frac{\Delta C_{\text{sig}}}{\Delta C_{\text{cal}}}, \quad T_{\text{cal}} = T_{\text{atm}} = T_{\text{gr}} = T_{\text{hot}}
\]

- No need to know $e^{-\tau_\nu}$ and $F_{\text{eff}}$ (Penzias & Burrus ARAA 1973):
- But $T_{\text{rec}}$ not known
The refined “Chopper Wheel” method

- General case: different $T_{\text{atm}}$, $T_{\text{hot}}$ and $T_{\text{gr}}$
  $\Rightarrow$ must solve for $e^{-\tau_{\nu}}$ and $F_{\text{eff}}$.
- $e^{-\tau_{\nu}}$: model of the atmosphere trying to reproduce $T_{\text{sky}}$ varying the amount of dominant species in the model (hence providing us with pwv).
- $F_{\text{eff}}$: skydips (measure atm. at several elevations)
- Perform $3 + 1$ measurements: hot, cold, empty sky, source

\[
C_{\text{sou}} = \chi \left\{ T_{\text{rec}} + T_{\text{sky}} + F_{\text{eff}} e^{-\tau_{\nu}} T_{\text{sou}} \right\}
\]
\[
C_{\text{atm}} = \chi \left\{ T_{\text{rec}} + T_{\text{sky}} \right\}
\]
\[
C_{\text{hot}} = \chi \left\{ T_{\text{rec}} + T_{\text{hot}} \right\}
\]
\[
C_{\text{col}} = \chi \left\{ T_{\text{rec}} + T_{\text{col}} \right\}
\]
Using hot & cold loads measurements lead to $T_{\text{rec}}$:

$$T_{\text{rec}} = \frac{T_{\text{hot}} - YT_{\text{col}}}{Y - 1}$$

$$Y = \frac{C_{\text{hot}}}{C_{\text{col}}} = \frac{T_{\text{rec}} + T_{\text{hot}}}{T_{\text{rec}} + T_{\text{col}}}$$

- $T_{\text{hot}} = 290 \text{ K}$
- $T_{\text{cold}} = 77 \text{ K}$
Calibration outputs (3): $T_{\text{sys}}$

*System temperature: describes the noise including all sources from the sky down to the backends*

- $T_{\text{sys}} = T_{\text{cal}} \frac{C_{\text{off}}}{\Delta C_{\text{cal}}}$
- used to determine the total statistical noise. For heterodyne receivers, noise is given by the “radiometer formula”:

\[
\sigma_T = \frac{\kappa \cdot T_{\text{sys}}}{\sqrt{\delta \nu \Delta t}}
\]

- $\delta \nu$: spectral resolution
- $\Delta t$: total integration time
- $\kappa$ depends on the observing mode:
- example: position switching, ON-OFF $\Rightarrow \sqrt{2}$,
  \[t_{\text{ON}} = t_{\text{OFF}} \Rightarrow \Delta t = 2t_{\text{ON}} \Rightarrow \sqrt{2}, \Rightarrow \kappa = 2\]
From $T_{mb}$ to $S_\nu$, from Kelvin to Jansky

- flux density: $S_\nu = \int_{\Omega} l_\nu(\Omega) \, d\Omega = \frac{2k}{\lambda^2} \int_{\Omega} T_R \, d\Omega$
- power received by the antenna: $kT'_A = k \frac{T^*_A}{F_{eff}} = \frac{1}{2} S_\nu A e$

$$\frac{S_\nu}{T^*_A} = \frac{2k}{A} \frac{F_{eff}}{\eta_A} \quad \text{Jy K}^{-1}$$

- $1 \text{ Jy} = 10^{-26} \text{ J s}^{-1} \text{ m}^{-2} \text{ Hz}^{-1}$
- values of $S_\nu / T^*_A$ are tabulated e.g. on IRAM-30m web page ($\approx 6 @ 100 \text{ GHz}, \approx 11 @ 340 \text{ GHz}$)
- How to convert the temperatures into $\text{ J s}^{-1} \text{ m}^{-2} \text{ Hz}^{-1} \text{ sr}^{-1}$?

$$l_\nu(\Omega) = \frac{2k}{\lambda^2} T_R(\Omega)$$
Summary
Image formation: total power telescope

- antenna scans the source
- image: convolution of $I_0$ by beam pattern $I'_{\nu} = \mathcal{P} \ast I_{0,\nu}$
- measure directly the brightness distribution $I_0$
Interferometer field of view

\[ F = D \ast (P \times I) + N \]

- **F** = dirty map = FT of observed visibilities
- **D** = dirty beam (\(-\rightarrow\) deconvolution)
- **P** = power pattern of single-dish
  *primary beam \(B\) in the following*
- **I** = sky brightness distribution
- **N** = noise distribution

- **An interferometer measures the product** \(P \times I\)
- **\(P\)** has a finite support \(\rightarrow\) limits the size of the field of view
Physical parameters

- Chose an adapted temperature scale \((T_A^*, T_{mb})\)
- Correct for error-beam pick-up when needed
- Amplitude calibration: often enough \(\rightarrow 10\%\) accuracy
Summary

- full-aperture antenna: $P \star I$
- interferometry sensitive to $P \times I$
- amplitude calibration:
  - converts counts into temperatures
  - corrects for atmospheric absorption
  - corrects for spillover
- lobe = main-lobe + error-lobes (e.g. as much as 50% in error-lobes at 230GHz for the 30m)
- Pay attention to the temperature scale to use ($T_A^*$, $T_{mb}$,...)
Image formation: correlation telescope

- Antennas fixed w.r.t. the source
- Correlation temperature: $T(0,0)$ Fourier transform of $I_0 \times P$
- Measure the Fourier transform of the brightness distribution $I_0$
- Image built afterwards
Interferometer field of view

Measurement equation of an interferometric observation:

\[ F = D \ast (B \times I) + N \]

- **F** = dirty map = FT of observed visibilities
- **D** = dirty beam (deconvolution)
- **B** = primary beam
- **I** = sky brightness distribution
- **N** = noise distribution

- **An interferometer measures the product** \( B \times I \)
- **B** has a finite support \( \rightarrow \) limits the size of the field of view
- **B** is a Gaussian \( \rightarrow \) primary beam correction possible (proper estimate of the fluxes) but strong increase of the noise
Primary beam width

Aperture function \( \Rightarrow \) Voltage pattern

\[
* \downarrow T(u, v) \downarrow |\cdot|^2
\]

Transfert function \( T(u, v) \) \( \Rightarrow \) Power pattern \( B(\ell, m) \)

= Primary beam

Gaussian illumination \( \Rightarrow \) to a good approximation, \( B \) is a Gaussian of \( 1.2 \frac{\lambda}{D} \) FWHM

Plateau de Bure

\( D = 15 \text{ m} \)

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Wavelength</th>
<th>Field of View</th>
</tr>
</thead>
<tbody>
<tr>
<td>85 GHz</td>
<td>3.5 mm</td>
<td>58''</td>
</tr>
<tr>
<td>100 GHz</td>
<td>3.0 mm</td>
<td>50''</td>
</tr>
<tr>
<td>115 GHz</td>
<td>2.6 mm</td>
<td>43''</td>
</tr>
<tr>
<td>215 GHz</td>
<td>1.4 mm</td>
<td>23''</td>
</tr>
<tr>
<td>230 GHz</td>
<td>1.3 mm</td>
<td>22''</td>
</tr>
<tr>
<td>245 GHz</td>
<td>1.2 mm</td>
<td>20''</td>
</tr>
</tbody>
</table>