V. Simplified bare pixels sensitivity calculations compared to a similar work by Desert

In "The case for a bolometric millimetre camera at the IRAM 30m telescope", Astro-ph 29 Jan 1999 and IRAM Newsletter Jan 1999, Désert & Benoit presented an estimation of bolometer performances for the 30m. Using the updated revision by Desert (2008), these performances are claculated again below and compared with a simplified version of my estimations to verify (1) the consistency of my procedure, (2) that with some approximations only few simple calculations are necessary, and (3) illustrate the effect of some variations in the input parameters.

Input parameters and hypothesis:

Desert : Telescope area: $A = 707\,m^2$ **Me:**

2 wavelengths $ii := 0..1$

$\lambda_D := \begin{pmatrix} 2.1 \\ 1.2 \end{pmatrix} mm$ $\nu_D := \frac{c}{\lambda_D}$ $\nu_D = \begin{pmatrix} 143 \\ 250 \end{pmatrix} GHz$ $\lambda_{ii+1} = \begin{pmatrix} 2.05 \\ 1.25 \end{pmatrix} mm$ $\nu_{ii+1} = \begin{pmatrix} 146 \\ 240 \end{pmatrix} GHz$

$\frac{\delta\nu}{\nu} := 0.30$ $\nu_{Dm} := \nu_D \cdot 0.85$ $\nu_{DM} := \nu_D \cdot 1.15$ $\frac{\delta\lambda}{\lambda} = \frac{2\,\nu_{ii+1} \cdot w_{ii+1}}{\left(\nu^2 - w^2\right)_{ii+1}} = \begin{pmatrix} 0.28 \\ 0.39 \end{pmatrix}$

$\delta\nu_D := \nu_{DM} - \nu_{Dm}$ $\delta\nu_D = \begin{pmatrix} 43 \\ 75 \end{pmatrix} GHz$ $2\,w_{ii+1} = \begin{pmatrix} 40 \\ 90 \end{pmatrix} GHz$

Writting the pixel size uFλ (see II.), the throughput $S\Omega=(\pi D^2/4)*((\pi/4)*(u*\lambda/D)^2)=(\pi/4)*u^2*\lambda^2$

Desert takes a pixel sampling a FWHM beam, (he uses the true factor 1.03, I approximate it with 1).

I take 0.5 Fλ for both bands, but with a 5% gap between pixels:

$u_{me} := 0.95 \cdot 0.5$ $u_{me} = 0.475$

$u_D := \begin{pmatrix} 0.5 \\ 0.5 \end{pmatrix}$ $S\Omega_{D_{ii}} := \left(\frac{\pi}{4}\right)^2 \cdot \left(1.03 \cdot u_{D_{ii}} \cdot \lambda_{D_{ii}}\right)^2$ $S\Omega_{me_{ii}} := \frac{\pi}{4} \cdot \left(0.475 \cdot \lambda_{ii+1}\right)^2$

D vs me:

$S\Omega_D = \begin{pmatrix} 0.72 \\ 0.24 \end{pmatrix} mm^2 sr$ = Ep = Eeff in Desert's sheet $\frac{\pi}{4} \cdot \left(\frac{1.03}{0.95}\right)^2 = 0.92$ $S\Omega_{me} = \begin{pmatrix} 0.74 \\ 0.28 \end{pmatrix} mm^2 sr$

$Ta_D := 250K$ $Tt_D := 280K$ $Tsup_D := 280K$ $T_{atm} = 270\,K$ $T_{tel} = 280\,K$ $Tf_{me} := 77K$

From my calculations in II.2., the dominant background is due to atmosphere and telescope, and in a lesser extent the 77K stage. In Desert the filter transmission is low compared to mine (see below), which means a higher emissivity thus a higher contribution, but he includes the filters in a global term P_{sup} including also mirrors, window and lens; at 280K with a 5% emissivity.

Transmission factors:

1) atmosphere

$\tau_{D1mm} := 0.35$ => ATM gives: $\tau_{D2mm} := 0.13$ $\tau\left(\nu_{ii+1}, 0\right) = \begin{pmatrix} 0.07 \\ 0.13 \end{pmatrix}$ $\tau\left(\nu_{ii+1}, 1\right) = \begin{pmatrix} 0.29 \\ 0.69 \end{pmatrix}$

Desert atmosphere is not great (~4mm wv), but between my 2 extreme choices

$ea_D := 1 - \exp\left[-\begin{pmatrix} \tau_{D2mm} \\ \tau_{D1mm} \end{pmatrix}\right]$ $ea_D = \begin{pmatrix} 0.12 \\ 0.30 \end{pmatrix}$ $ea_{me_{ii,v}} := ea\left(\nu_{ii+1}, v\right)$ $ea_{me} = \begin{pmatrix} 0.07 & 0.25 \\ 0.12 & 0.50 \end{pmatrix}$

Remark: though T_{atm}-Ta_D=20K, for identical contents of water vapor my ta_{me} is few % < ta_D because Desert's τ are at Zenith whereas mine are corrected for 50 degree elevation.

$et_D := 0.1$ 2) telescope emissivity $et = 0.11$

3) filters + 77K lens

I choose $nfb = 7$ filters stages plus the band pass filter plus 1 lens, each with a transmission $tf = 95\%$ but only $nf77 = 3$ on the 77K stage while the others are at colder stages.

Desert assumes a warm lens, I find [thesis] e(2cm)=5%, but he took e(5cm)=15%, (my t_{fD} = his tfilt*tlen)

$tf_{me} := tf^{nfb+1}$ $tf_{me} = 0.66$

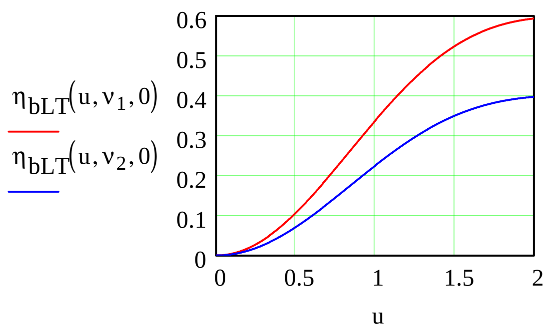
$tf_D := 0.15 \cdot 0.85$ $tf_D = 0.13$ $ef_D := 1 - tf_D$ $ef_D = 0.87$ $ef_{me} := 1 - tf^{nf77}$ $ef_{me} = 0.14$

0.15 vs 0.66 ==> Desert filters are really bad compared to Cardiff's specifications !

If I would use filters as bad as Desert's, I would have to choose Tf<<10K to avoid Pf dominates the background ! Desert avoid the problem introducing an artificial esup = 5% ; suspicious (see below) !

4) efficiency linked to the diffraction pattern

The figure opposite shows the integral of the diffraction pattern up to the pixel size (u) in unit of Fλ (=> η_{bLT} = pixel efficiency). The red line is for the 2mm band and the blue line for the 1mm band. u=0.5 is required for Nyquist sampling and gives η_{pix}. u=1 is for beam sampling (= FWHM). u=2 is the full beam (1st dark ring) and gives η_{Beff}.



The only efficiency related to the diffraction pattern in Desert's Excel sheet is his main lobe efficiency: $EffLP := \begin{pmatrix} 0.50 \\ 0.25 \end{pmatrix}$

pixel: $\eta_{pixme_{ii}} := \eta_{bLT}(rs \cdot u_{me}, \nu_{ii+1}, 0)$

FWHM: $\eta_{HPme_{ii}} := \eta_{bLT}(rs \cdot 1 \cdot 0.95, \nu_{ii+1}, 0)$

1st ring: $\eta_{Beffme_{ii}} := \eta_{bLT}(rs \cdot 2 \cdot 0.95, \nu_{ii+1}, 0)$

Usually "main lobe" refers to the full beam (2Fλ), it seems this is what he means in his calculation of the power from a point source (see the factor EffLP/5 below), but his calculation of the NEFD suggests he rather means FWHM efficiency ("4 pixels per beam") !

$\eta_{pixme} = \begin{pmatrix} 0.13 \\ 0.09 \end{pmatrix}$ $\eta_{HPme} = \begin{pmatrix} 0.38 \\ 0.26 \end{pmatrix}$ $\eta_{Beffme} = \begin{pmatrix} 0.60 \\ 0.40 \end{pmatrix}$

For the 2mm band EffLP is between my Half Power efficiency and beam 1st dark ring efficiency, whereas for the 1mm band EffLP = η_{HPme}. Does this mean that one of Desert's values is wrong ?

5) others leakage & blockage $to = 0.97$

Apparently no other factors for D&B

detector quantum efficiency $\eta_d = 0.90$

forward efficiency $F_{eff_{ii+1}} = \begin{pmatrix} 0.90 \\ 0.86 \end{pmatrix}$

In his 2008 revision, Desert added calculations for the same λ as me, with filters closer to mine tf_{Dnew}=0.7, a revised EffLP that I cite below as B_{eff} because the values come from Beff in Greve's article "The beam pattern of the IRAM 30m telescope", and the other factors follow the same rule as his "initial case". The ratios of **his "best case" factors versus my transmission factors**:

λ:	SΩ:	t atmo:	B_{eff}:	telescope + lens filters + others (but without F_{eff}):	Global ratio without B_{eff}:
2.05mm	$\frac{69}{74} = 0.93$	$\frac{0.96}{1-0.07} = 1.03$	$\frac{0.54}{0.60} = 0.90$		$\frac{1.03^2 \cdot 0.93}{F_{eff_{ii+1}}} = \begin{pmatrix} 1.10 \\ 1.15 \end{pmatrix}$
1.25mm	$\frac{26}{28} = 0.93$	$\frac{0.91}{1-0.12} = 1.03$	$\frac{0.42}{0.40} = 1.05$	$\frac{0.7 \cdot 0.85}{0.66 \cdot 0.97 \cdot 0.9} = 1.03$	

==> Desert's "best case" global transmission is comparable to mine, whereas the match with his "initial case" is really poor (global ratio D/me=0.24) becausse of his bad filters. Keeping the bad filters for the continuation of the comparison below will bring an interesting result.

6) Global factors for atmosphere, telescope and cryostat optics

$$eta_D := ea_D \cdot \left(1 - et_D\right) \cdot tf_D$$

$$ett_D := et_D \cdot tf_D$$

$$eta_{me_{ii,v}} := ea_{me_{ii,v}} \cdot (1 - et) \cdot tf_{me} \cdot \left(to \cdot \eta_d \cdot F_{eff_{ii+1}}\right)$$

$$ett_{me_{ii}} := et \cdot tf_{me} \cdot \left(to \cdot \eta_d \cdot F_{eff_{ii+1}}\right)$$

Desert has no direct contribution of filters, but an artificial e=5% warm optics contribution:

$$etf_{me} := ef_{me} \cdot tf^{\,nfb-nf77}$$

$$etsup_D := tf_D \cdot 0.05 \qquad etsup_D = 0.64\,\%$$

$$tsky_D := \left(1 - ea_D\right) \cdot \left(1 - et_D\right) \cdot tf_D$$

$$tsky_{me_{ii,v}} := \left(1 - ea_{me_{ii,v}}\right) \cdot (1 - et) \cdot tf_{me} \cdot \left(to \cdot \eta_d \cdot F_{eff_{ii+1}}\right)$$

$$eta_D = \begin{pmatrix} 1.4 \\ 3.4 \end{pmatrix} \% \qquad ett_D = 1.3\,\% \qquad etsup_D = 0.6\,\%$$

$$tsky_D = \begin{pmatrix} 10 \\ 8 \end{pmatrix} \%$$

$$eta_{me} = \begin{pmatrix} 3.3 & 11.9 \\ 5.3 & 22.3 \end{pmatrix} \% \qquad ett_{me} = \begin{pmatrix} 5.5 \\ 5.3 \end{pmatrix} \% \qquad etf_{me} = 11.6\,\%$$

$$tsky_{me} = \begin{pmatrix} 43 & 35 \\ 39 & 22 \end{pmatrix} \%$$

<u>Heat load from background power:</u>

Recall of Plank black body brightness : $Br(T,\nu) := \frac{2 \cdot h}{c^2} \cdot \frac{\nu^3}{\exp\left(\frac{h \cdot \nu}{k \cdot T}\right) - 1}$

Assuming the product SΩ(ν)*et(ν)*Br(ν) is roughly linear in the bandwidth δν, its integration can be approximated with the product SΩ(ν_c)*et(ν_c)*Br(ν_c)*δν where ν_c is the band center, thus giving simple formulas for the power deposited on the detectors:

$$P_{aD_{ii}} := S\Omega_{D_{ii}} \cdot eta_{D_{ii}} \cdot Br\left(Ta_D, \nu_{D_{ii}}\right) \cdot \delta\nu_{D_{ii}}$$

$$P_{ame_{ii,v}} := S\Omega_{me_{ii}} \cdot eta_{me_{ii,v}} \cdot Br\left(T_{atm}, \nu_{ii+1}\right) \cdot 2 \cdot w_{ii+1}$$

$$P_{tD_{ii}} := S\Omega_{D_{ii}} \cdot ett_D \cdot Br\left(Tt_D, \nu_{D_{ii}}\right) \cdot \delta\nu_{D_{ii}}$$

$$P_{tme_{ii}} := S\Omega_{me_{ii}} \cdot ett_{me_{ii}} \cdot Br\left(T_{tel}, \nu_{ii+1}\right) \cdot 2 \cdot w_{ii+1}$$

$$P_{supD_{ii}} := S\Omega_{D_{ii}} \cdot etsup_D \cdot Br\left(Tsup_D, \nu_{D_{ii}}\right) \cdot \delta\nu_{D_{ii}}$$

$$P_{fme_{ii}} := S\Omega_{me_{ii}} \cdot etf_{me} \cdot Br\left(Tf_{me}, \nu_{ii+1}\right) \cdot 2 \cdot w_{ii+1}$$

$$P_{TD} := P_{aD} + P_{tD} + P_{supD}$$

$$P_{Tme_{ii,v}} := P_{ame_{ii,v}} + P_{tme_{ii}} + P_{fme_{ii}}$$

$P_{aD} = \begin{pmatrix} 0.67 \\ 2.80 \end{pmatrix} \text{pW}$ $\quad P_{tD} = \begin{pmatrix} 0.68 \\ 1.18 \end{pmatrix} \text{pW}$ $\quad P_{ame} = \begin{pmatrix} 1.7 & 6.2 \\ 6.2 & 26.0 \end{pmatrix} \text{pW}$ $\quad P_{tme} = \begin{pmatrix} 3.0 \\ 6.4 \end{pmatrix} \text{pW}$

$P_{supD} = \begin{pmatrix} 0.3 \\ 0.6 \end{pmatrix} \text{pW}$ $\quad \boxed{P_{TD} = \begin{pmatrix} 1.7 \\ 4.6 \end{pmatrix} \text{pW}}$ $\quad P_{fme} = \begin{pmatrix} 1.7 \\ 3.7 \end{pmatrix} \text{pW}$ $\quad \boxed{P_{Tme} = \begin{pmatrix} 6.4 & 10.8 \\ 16.2 & 36.0 \end{pmatrix} \text{pW}}$

Desert's sheet using my Thesis polynomial approximation for atmospheric transmission:

$P_{ads} := \begin{pmatrix} 0.77 \\ 3.05 \end{pmatrix} \text{pW}$ $\quad$ His total: $P_{Tds} := \begin{pmatrix} 1.9 \\ 4.9 \end{pmatrix} \text{pW}$

My values with the exact calculations from III.2.:

$$P_{Tm_{ii,v}} := P_{ab_{ii+1,v}} + P_{tb_{ii+1}} + P_{77b_{ii+1}}$$

$$P_{Tm} = \begin{pmatrix} 6.3 & 10.7 \\ 16.5 & 36.7 \end{pmatrix} \text{pW}$$

My results are consistent with III.2. ==> the linear approximation of the spectral power in the bandwidth is correct (and P_{tD} 1%< Desert's Excel sheet 3rd oder polynome approximation of Br).

- Atmosphere: $|P_{aD}-P_{apD}|>0$ comes from the transmission (opacity) model ; Desert uses a 3rd order polynome fit for each band (from my thesis), more precise than the global multiband fit I defined in III.1. Beside the atmosphere model, the other transmission factors contribute to the difference between Desert's sheet and P_{ame}.
- Telescope: the transmission factors explain the difference between Desert's sheet and me.
- Cryostat optics: Desert's P_{sup} using his "best case" transmission factors (0.7 for the filters) is ~= P_{fme}, though calculated with different hypothesis, mostly because his T_{sup} > to my T_f is compensated by his low "suplementary" emissivity (esup=5%). But using the very low transmission of his "initial case" filters (0.15) in my method gives a very big filter emissivity, thus a very strong totally dominant and unrealistic contribution of the filters to the background.

Beside these differences, our calculations are consistent with each other so far.

Remark: my optics description [77K + 4K + cold parts] seems more realistic than Desert's low transmission, low emissivity, warm supplementary component. Nevertheless my method assumes e+t=1, hence reflection is neglected. For the mirrors reflection = transmission of the signal to the cryostat (so it's OK), but for the filters the reflection is back to the sky. Thus P_{fme} may be actually lower than my calculation, but the filters transmission shouldn't be changed, hence neither the effect on the other background sources, so I still prefer my method.

<u>Typical sources power</u>

Extended source: $\quad T_{RJt} := 1\text{K} \quad B_{RJt}(\lambda) := \dfrac{2\cdot k\cdot T_{RJt}}{\lambda^2}$

$$P_{rjD_{ii}} := S\Omega_{D_{ii}}\cdot tsky_{D_{ii}}\cdot B_{RJt}\left(\lambda_{D_{ii}}\right)\cdot \delta\nu_{D_{ii}}$$

$$P_{rjme_{ii,v}} := S\Omega_{me_{ii}}\cdot tsky_{me_{ii,v}}\cdot B_{RJt}\left(\lambda_{ii+1}\right)\cdot 2\cdot w_{ii+1}$$

$P_{rjD} = \begin{pmatrix} 19.5 \\ 27.4 \end{pmatrix} \text{fW}$ $\quad P_{rjme} = \begin{pmatrix} 85 & 68 \\ 173 & 98 \end{pmatrix} \text{fW}$

From III.2. @ 1mmwv:

$$P_{RJb_{ii+1,0}} = \begin{pmatrix} 84 \\ 175 \end{pmatrix} \text{fW}$$

With EffLP: $EffLP_{ii}\cdot P_{rjD_{ii}} = \begin{pmatrix} 9.8 \\ 6.8 \end{pmatrix} \text{fW}$

~small difference with his sheet due to 3rd order approx in band integration in his calculation.

my value corrected with the differences in transmissions from Desert's Excel sheet and including his EffLP for my λ: $1.10 \cdot 0.54 \cdot 85 = 50.5$ / $1.15 \cdot 0.42 \cdot 173 = 83.6$ ~ same as his sheet

Desert included the beam efficiency (EffLP) as a "pixel efficiency" for extended sources, but the convolution of diffraction beam with the source shape says that if the source image is bigger than the diffraction pattern, the illumination is mostly uniform, so F_{eff} must be used, not B_{eff} !

Point source: $F_t := 1\text{mJy}$

Using the same method as Desert's sheet:

$$P_{ptD_{ii}} := \frac{\text{EffLP}_{ii}}{5} \cdot \text{tsky}_{D_{ii}} \cdot A \cdot F_t \cdot \delta\nu_{D_{ii}}$$

$$P_{ptD} = \begin{pmatrix} 0.31 \\ 0.21 \end{pmatrix} 10^{-17} \text{W}$$ ~same as his sheet

The factor EffLP/5 comes from the argument that 1/5 of the power is in the pixel, which is compatible with :

$$\frac{\eta_{\text{Beffme}_{ii}}}{\eta_{\text{pixme}_{ii}}} = \begin{pmatrix} 4.7 \\ 4.7 \end{pmatrix}$$

$$P_{ptme_{ii,v}} := \frac{\eta_{\text{pixme}_{ii}}}{F_{eff_{ii+1}}} \cdot \text{tsky}_{me_{ii,v}} \cdot A \cdot F_t \cdot 2 \cdot w_{ii+1}$$

$$P_{ptme} = \begin{pmatrix} 1.7 & 1.4 \\ 2.5 & 1.4 \end{pmatrix} 10^{-17} \text{W}$$

Values from III.2. @ 1mmwv: $P_{ptob_{ii+1,0}} = \begin{pmatrix} 1.7 \\ 2.5 \end{pmatrix} 10^{-17} \text{W}$

=> Desert's method suggests that EffLP is the (full) beam efficiency, which is apparently confirmed by his "best case" values for EffLP.

==> Apart the problem of EffLP (B_{eff}) in Desert's calculation for the 1KRJ extended source, and the very bad filters of his "initial case", Desert's and my calculations of powers are consistent, with only some minor differences in the transmission factors.

<u>Photon noise (approximations):</u>

Shot noise (Poissonian): $NEP_{TOT}^2 = \Sigma NEP^2$

$$\text{NEPp}_{TD_{ii}} := \sqrt{2 \cdot h \cdot \nu_{D_{ii}} \cdot P_{TD_{ii}}}$$

$$\text{NEPp}_{Tme_{ii,v}} := \sqrt{2 \cdot h \cdot \nu_{ii+1} \cdot P_{Tme_{ii,v}}}$$

Bunching noise (boson): $NEP_{TOT} = \Sigma NEP$

Incoherent beam approximation (spacial coherence: $\Delta \sim \lambda^2/A\Omega$):

$$\text{NEPbi}_{TD_{ii}} := \sqrt{2 \cdot k} \cdot \left(\sqrt{\text{Ta}_D \cdot \text{eta}_{D_{ii}} \cdot P_{aD_{ii}}} + \sqrt{\text{Tt}_D \cdot \text{ett}_D \cdot P_{tD_{ii}}} + \sqrt{\text{Tsup}_D \cdot \text{etsup}_D \cdot P_{supD_{ii}}} \right)$$

$$\text{NEPbi}_{Tme_{ii,v}} := \sqrt{2 \cdot k} \cdot \left(\sqrt{T_{atm} \cdot \text{eta}_{me_{ii,v}} \cdot P_{ame_{ii,v}}} + \sqrt{T_{tel} \cdot \text{ett}_{me_{ii}} \cdot P_{tme_{ii}}} + \sqrt{\text{Tf}_{me} \cdot \text{etf}_{me} \cdot P_{fme_{ii}}} \right)$$

Coherent beam approximation ($\Delta \sim 1$):

$$\text{NEPbc}_{TD_{ii}} := \frac{P_{aD_{ii}} + P_{tD_{ii}} + P_{supD_{ii}}}{\sqrt{\delta\nu_{D_{ii}}}}$$

$$\text{NEPbc}_{Tme_{ii,v}} := \frac{P_{ame_{ii,v}} + P_{tme_{ii}} + P_{fme_{ii}}}{\sqrt{2 \cdot w_{ii+1}}}$$

Numerical application:

Convenient noise unit: $nu = 1 \times 10^{-17} \frac{W}{\sqrt{Hz}}$

$NEP_{pame_{ii,v}} := \sqrt{2 \cdot h \cdot \nu_{ii+1} \cdot P_{ame_{ii,v}}}$

$\sqrt{2 \cdot h \cdot \nu_{D_{ii}} \cdot P_{aD_{ii}}} = \begin{pmatrix} 1.12 \\ 3.05 \end{pmatrix} nu$

$NEP_{pame} = \begin{pmatrix} 1.83 & 3.46 \\ 4.44 & 9.08 \end{pmatrix} nu$

$\sqrt{2 \cdot h \cdot \nu_{D_{ii}} \cdot P_{tD_{ii}}} = \begin{pmatrix} 1.14 \\ 1.98 \end{pmatrix} nu$

$\sqrt{2 \cdot h \cdot \nu_{ii+1} \cdot P_{tme_{ii}}} = \begin{pmatrix} 2.40 \\ 4.49 \end{pmatrix} nu$

$\sqrt{2 \cdot h \cdot \nu_{D_{ii}} \cdot P_{supD_{ii}}} = \begin{pmatrix} 0.80 \\ 1.40 \end{pmatrix} nu$

$\sqrt{2 \cdot h \cdot \nu_{ii+1} \cdot P_{fme_{ii}}} = \begin{pmatrix} 1.80 \\ 3.41 \end{pmatrix} nu$

Desert's sheet (index 1 is for poisson, 2 is for boson):

$NEP_{aD1} := \begin{pmatrix} 1.22 \\ 3.22 \end{pmatrix} \cdot nu$ $\quad NEP_{tD1} := \begin{pmatrix} 1.15 \\ 2.00 \end{pmatrix} \cdot nu$

$NEP_{supD1} := \begin{pmatrix} 1.44 \\ 2.51 \end{pmatrix} \cdot nu$

My exact calculs from III.4.:

$NEP_{pam_{ii,v}} := NEP_{pab_{ii+1,v}}$ $\quad NEP_{pam} = \begin{pmatrix} 1.8 & 3.5 \\ 4.6 & 9.3 \end{pmatrix} nu$

$NEP_{ptb_{ii+1}} = \begin{pmatrix} 2.4 \\ 4.5 \end{pmatrix} nu$ $\quad NEP_{p77b_{ii+1}} = \begin{pmatrix} 1.8 \\ 3.3 \end{pmatrix} nu$

=> In the bandwidth integration, the approximation of the spectral power by its value at band center gives correct results (errors are only few %).

NEP_a: difference due to opacity model (see P)

NEP_t: OK (linear vs 3rd order approximation)

NEP_{sup}: Desert's value from my old Excel sheet: with **esup=0.02** (not 0.05) and **without tfilt ! => not compatible with his P_{sup} !** Using esup=0.05 and tfilt=0.15 in his Excel formula => results ~= calcul above (same difference as telescope case).

$\frac{P_{aD_{ii}}}{\sqrt{\delta\nu_{D_{ii}}}} = \begin{pmatrix} 0.32 \\ 1.02 \end{pmatrix} nu$ $\quad \sqrt{2 \cdot k \cdot Ta_D \cdot eta_{D_{ii}} \cdot P_{aD_{ii}}} = \begin{pmatrix} 0.80 \\ 2.56 \end{pmatrix} nu$

$NEP_{bame_{ii,v}} := \frac{P_{ame_{ii,v}}}{\sqrt{2 \cdot w_{ii+1}}}$

$\frac{P_{tD_{ii}}}{\sqrt{\delta\nu_{D_{ii}}}} = \begin{pmatrix} 0.33 \\ 0.43 \end{pmatrix} nu$ $\quad \sqrt{2kTt_D \cdot ett_D \cdot P_{tD_{ii}}} = \begin{pmatrix} 0.82 \\ 1.08 \end{pmatrix} nu$

$NEP_{bame} = \begin{pmatrix} 0.86 & 3.09 \\ 2.07 & 8.65 \end{pmatrix} nu$

$\frac{P_{supD_{ii}}}{\sqrt{\delta\nu_{D_{ii}}}} = \begin{pmatrix} 0.16 \\ 0.22 \end{pmatrix} nu$ $\quad \sqrt{2 \cdot k} \cdot \left(\sqrt{Tsup_D \cdot etsup_D \cdot P_{supD_{ii}}}\right) = \begin{pmatrix} 0.41 \\ 0.54 \end{pmatrix} nu$

$\frac{P_{tme_{ii}}}{\sqrt{2 \cdot w_{ii+1}}} = \begin{pmatrix} 1.49 \\ 2.12 \end{pmatrix} nu$

$\frac{P_{fme_{ii}}}{\sqrt{2 \cdot w_{ii+1}}} = \begin{pmatrix} 0.84 \\ 1.22 \end{pmatrix} nu$

Desert's sheet (index 2 is for boson):

$NEP_{aD2} := \begin{pmatrix} 0.54 \\ 1.65 \end{pmatrix} \cdot nu$ $\quad NEP_{tD2} := \begin{pmatrix} 0.48 \\ 0.62 \end{pmatrix} \cdot nu$

$NEP_{supD2} := \begin{pmatrix} 0.75 \\ 0.98 \end{pmatrix} \cdot nu$

My exact calculs from III.4.:

$NEP_{bam_{ii,v}} := NEP_{bab_{ii+1,v}}$ $\quad NEP_{bam} = \begin{pmatrix} 0.8 & 2.7 \\ 1.9 & 7.9 \end{pmatrix} nu$

a $\sqrt{2}$ factor is need to find my result:

$\frac{NEP_{tD2}}{\sqrt{2}} = \begin{pmatrix} 0.34 \\ 0.44 \end{pmatrix} nu$

$NEP_{btb_{ii+1}} = \begin{pmatrix} 1.3 \\ 1.9 \end{pmatrix} nu$ $\quad NEP_{b77b_{ii+1}} = \begin{pmatrix} 0.7 \\ 1.0 \end{pmatrix} nu$

Desert's NEP are compatible with the coherent approximation using Δ=1, but for a $\sqrt{2}$ **factor** ! This comes from my old excel sheet and **supposed** that the incoming **powers were all polarised** (dp=1 => po=2, instead of po=1 for unpolarized beam, see III.2), but from the calculations of P this is not the case ! => the $\sqrt{2}$ factor in Desert sheet is a mistake !

=> NEP_{bme} are bigger than exact calculations because the correct spacial coherence for 0.5Fλ pixels is Δ=0.8 (not 1 as in the approximation).

Total photon noise:

$$NEPp_{TD} = \begin{pmatrix} 1.8 \\ 3.9 \end{pmatrix} nu \qquad NEPp_{Tme} = \begin{pmatrix} 3.5 & 4.6 \\ 7.2 & 10.7 \end{pmatrix} nu$$

$$NEPbi_{TD} = \begin{pmatrix} 2.0 \\ 4.2 \end{pmatrix} nu \qquad NEPbi_{Tme} = \begin{pmatrix} 7.7 & 13.0 \\ 13.1 & 28.9 \end{pmatrix} nu$$

$$NEPbc_{TD} = \begin{pmatrix} 0.8 \\ 1.7 \end{pmatrix} nu \qquad NEPbc_{Tme} = \begin{pmatrix} 3.2 & 5.4 \\ 5.4 & 12.0 \end{pmatrix} nu$$

$$NEP_{TD} := \sqrt{NEPp_{TD}^{\;2} + NEPbc_{TD}^{\;2}} \qquad NEP_{Tme_{ii,v}} := \sqrt{\left(NEPp_{Tme_{ii,v}}\right)^2 + \left(NEPbc_{Tme_{ii,v}}\right)^2}$$

$$\boxed{NEP_{TD} = \begin{pmatrix} 2.0 \\ 4.2 \end{pmatrix} 10^{-17} \frac{W}{\sqrt{Hz}}} \qquad \boxed{NEP_{Tme} = \begin{pmatrix} 4.7 & 7.1 \\ 9.0 & 16.1 \end{pmatrix} 10^{-17} \frac{W}{\sqrt{Hz}}}$$

Desert 10/2008 Excel sheet (4mm wv):

P_{tot} method
(shot noise only)

$$NEP_{pTds} := \begin{pmatrix} 1.9 \\ 4.0 \end{pmatrix} \cdot 10^{-17} \frac{W}{\sqrt{Hz}}$$

Total from each shot noise component:

$$\sqrt{NEP_{aD1}^{\;2} + NEP_{tD1}^{\;2} + NEP_{supD1}^{\;2}} = \begin{pmatrix} 2.2 \\ 4.5 \end{pmatrix} nu$$

not = NEP_{pTds} because of the error in NEP_{supD1}

$$NEP_{Tds} := \begin{pmatrix} 2.8 \\ 5.6 \end{pmatrix} \cdot 10^{-17} \frac{W}{\sqrt{Hz}}$$

=> Desert's NEP_{Tds} > NEP_{TD} because of the error in NEP_{sup} and the $\sqrt{2}$ factor from polarisation in the bunching noise components (the effect of different fits for atmosphere opacity is much smaller)

My results from III.4.:

$$NEP_{Tm_{ii,v}} := NEP_{phTb_{ii+1,v}}$$

$$NEP_{Tm} = \begin{pmatrix} 4.9 & 7.0 \\ 9.5 & 16.1 \end{pmatrix} 10^{-17} \cdot \frac{W}{\sqrt{Hz}}$$

=> My results are totally consistent with exact calculations in III.4. and III.5. **The calculations from Desert's 2008 Excel sheet tend to converge toward mine** (compared to his early work with Benoit), **but there's errors in Desert's sheet remnant from my thesis calculations...**

<u>Optimal pixel intrinsic noise</u>

Desert imposes the constraint NEP_{int}=NEP_T/2 (as in my thesis). But the comparison NEP_{TD} vs NEP_{Tme} show that without an excelent knowledge of the future instrument optical chain transmission, **the safest attitude to make sure the detector intrinsic noise will be nigligible compared to the background is to impose NEP_{int} = min(NEP_T)/6** (not NEP_T/3 as expected from the value of NEP_{Tme} only) !

$$NEP_{Dint} := \frac{NEP_{TD}}{2}$$ $$\boxed{NEP_{Dint} = \begin{pmatrix} 1.0 \\ 2.1 \end{pmatrix} 10^{-17} \frac{W}{\sqrt{Hz}}}$$ $$\boxed{NEP_{bare_{ii+1}} = \begin{pmatrix} 0.8 \\ 1.6 \end{pmatrix} 10^{-17} \frac{W}{\sqrt{Hz}}}$$

Though obtained with somewhat different hypothesis and reasonings the instrumental noise from Desert's Excel sheet is only a factor 2 above mine, so they are by chance roughly consistent.

TOTAL optimal noise

$$NEP_D := NEP_{TD} \cdot \sqrt{1 + 0.5^2}$$

$$NEP_{me_{ii,v}} := \sqrt{\left(NEP_{Tme_{ii,v}}\right)^2 + \left(NEP_{bare_{ii+1}}\right)^2}$$

$$\boxed{NEP_D = \begin{pmatrix} 2.2 \\ 4.7 \end{pmatrix} 10^{-17} \frac{W}{\sqrt{Hz}}}$$

$$\boxed{NEP_{me} = \begin{pmatrix} 4.8 & 7.1 \\ 9.1 & 16.1 \end{pmatrix} 10^{-17} \frac{W}{\sqrt{Hz}}}$$

Desert 10/2008 Excel sheet (4mm wv): $$NEP_{ds} := \begin{pmatrix} 3.2 \\ 6.3 \end{pmatrix} 10^{-17} \frac{W}{\sqrt{Hz}}$$

results from III.4.: see NEP_{Tme} vs NEP_{Tm}

Co-addition of pixels

Consistency of physics : the effect of a physical environment on a system does not depend on the way the observer consider this system, in particular either as a whole or made of N subsystems. Thus N coadded pixels must be equivalent to 1 pixel the same size as them.

Only the throughput changes when scaling from a 0.5Fλ pixel to a 1Fλ: $S\Omega_{1F\lambda} = 4*S\Omega_{0.5F\lambda}$.
Desert assumes NEP is poissonian and varies as $\sqrt{N}$, where N is the number of coadded pixels.

But the bunching noise NEPb~$\sqrt{\Delta \cdot P}$, and in the 0.5..4Fλ range Δ~1/$\sqrt{N}$ so NEPb~$N^{3/4}$ (see II.3).

Does the fact that NEPb varies with $N^{3/4}$ mean that pixels are not independent ? Is it the effect of inter pixel correlation and is it equivalent to the covariance matrix in Zmuidzinas ?

number of pixels to coadd: $Nb := 4$

$$2\frac{3}{4} = 1.5$$

$$NEP_{ND} := \sqrt{Nb \cdot \left(NEPp_{TD}^2 + NEP_{Dint}^2\right) + Nb^{1.5} \cdot NEPbc_{TD}^2}$$

$$NEP_{Nme_{ii,v}} := \sqrt{Nb \cdot \left[\left(NEPp_{Tme_{ii,v}}\right)^2 + \left(NEP_{bare_{ii+1}}\right)^2\right] + Nb^{1.5} \cdot \left(NEPbc_{Tme_{ii,v}}\right)^2}$$

$$\boxed{NEP_{ND} = \begin{pmatrix} 4.7 \\ 10.0 \end{pmatrix} 10^{-17} \frac{W}{\sqrt{Hz}}}$$

$$\boxed{NEP_{Nme} = \begin{pmatrix} 11.5 & 17.9 \\ 21.2 & 40.2 \end{pmatrix} 10^{-17} \frac{W}{\sqrt{Hz}}}$$

Impossible to define a general formula for RN=NEP_N/NEP because it depends on NEPp/NEPb, only particular cases corresponding to the values calculated previously can be calculated:

NEPp=2NEPb:

$$RN_D := \sqrt{\frac{1}{13} \cdot \left(11 \cdot Nb + 2 \cdot Nb^{1.5}\right)}$$

$$R4_D := 2.1$$

$$R16_D := 4.8$$

$$RN_D \cdot NEP_D = \begin{pmatrix} 4.7 \\ 10.2 \end{pmatrix} nu$$

NEPp=NEPb:

$$RN_{me} := \sqrt{\frac{1}{2} \cdot \left(Nb + Nb^{1.5}\right)}$$

$$R4_{me} := 2.4$$

$$R16_{me} := 6.3$$

$$RN_{me} \cdot NEP_{me} = \begin{pmatrix} 11.8 & 17.5 \\ 22.3 & 39.5 \end{pmatrix} nu$$

From III.4. the NEP calculated for a 1Fλ pixel (same size as 4*0.5Fλ pixels) is:

$$NEP_{1F\lambda_{ii,v}} := NEP_{Tb1F\lambda_{ii+1,v}} \qquad NEP_{1F\lambda} = \begin{pmatrix} 12.0 & 18.0 \\ 23.0 & 41.0 \end{pmatrix} nu$$

=> The calculation of co-addition as a scalling to a bigger pixel is consistent at few %.

Co-addition is useful to check the calculation of NEP is coherent with variation of pixel size, besides it will help defining convenient sensitivity parameters (NET & NEFD) independent pixel.

Remark: my transmission criterias and the blackbody properties of the background components implies that the constraint on the instrumental noise is $R4_{me}/2$=1.2 less demanding for a $(1F\lambda)^2$ area than expected from "poissonian" scalling of 0.5Fλ pixels. Hence the bunching noise implies that it is more (less) demanding to build small (big) pixels than expected from shot noise only !

Noise Equivalent Temperature Density (NET):

$$\mu K := 10^{-6} K$$

1 pixel:

General formula compatible with Desert's formulation:

$$NET := \frac{NEP \cdot T_{RJ}}{\sqrt{2} \cdot P_{RJ}}$$

Replacing P_{RJ} with its 1st order approximation:

$$NET := \frac{NEP \cdot \lambda^2 \cdot po}{\sqrt{2} \cdot S\Omega \cdot t_{sky} \cdot 2k \cdot \Delta\nu}$$

The $\sqrt{2}$ factor comes from the sampling frequency and is introduced to make the defined quantity proportional to the integration time. This is stressed in the units by the use of $\sqrt{s}$ instead of $\sqrt{Hz}$ (in other words $\sqrt{Hz}$ refers to the quantity before sampling and $\sqrt{s}$ refers to the recorded data).

$$NET_{D_{ii}} := \frac{NEP_{D_{ii}} \cdot T_{RJt}}{\sqrt{2} \cdot P_{rjD_{ii}}} \qquad NET_D = \begin{pmatrix} 797 \\ 1223 \end{pmatrix} \mu K \cdot \sqrt{s}$$

$$NET_{me_{ii,v}} := \frac{NEP_{me_{ii,v}} \cdot T_{RJt}}{\sqrt{2} \cdot P_{rjme_{ii,v}}}$$

Desert's sheet:

$$\frac{NET_{D_{ii}}}{EffLP_{ii}} \cdot \frac{NEP_{ds_{ii}}}{NEP_{D_{ii}}} = \begin{pmatrix} 2321 \\ 6506 \end{pmatrix} \mu K \cdot \sqrt{s}$$

~ close to his values

$$NET_{me} = \begin{pmatrix} 401 & 742 \\ 373 & 1164 \end{pmatrix} \mu K \cdot \sqrt{s}$$

=> Desert's NET inherit the problems from EffLP in his P_{RJ} and the wrong P_{sup} in his NEP. When introducing these "factors" in the calculation, a small difference with the results of his sheet remains, it is due to the 3rd order approx he uses for the band integral.

The source temperature is proportional to the brightness = flux/steradians, which is the quantity to measure for extended sources. For this reason the **NET is often used as a parameter of sensitivity to extended sources**. But since its value depends on pixel size, the universality of such parameter can only exist if it is defined for a standarised size: the FWHM beam or full beam are usually used. Desert uses the FWHM beam (4 pixels), like I do.

Nb coadded pixels:

$$NET_{ND_{ii}} := \frac{NEP_{ND_{ii}} \cdot T_{RJt}}{\sqrt{2} \cdot Nb\, P_{rjD_{ii}}} \qquad NET_{Nme_{ii,v}} := \frac{NEP_{Nme_{ii,v}} \cdot T_{RJt}}{\sqrt{2} \cdot Nb\, P_{rjme_{ii,v}}}$$

$$\boxed{NET_{ND} = \begin{pmatrix} 425 \\ 648 \end{pmatrix} \mu K \cdot \sqrt{s}} \qquad \boxed{NET_{Nme} = \begin{pmatrix} 241 & 466 \\ 217 & 725 \end{pmatrix} \mu K \cdot \sqrt{s}}$$

Scaling for 4 and 16 pixels

$$\frac{4}{R4_D} = 1.9 \qquad \frac{NET_D}{1.9} = \begin{pmatrix} 420 \\ 644 \end{pmatrix} \mu K \cdot \sqrt{s} \qquad \frac{4}{R4_{me}} = 1.7 \qquad \frac{NET_{me}}{1.7} = \begin{pmatrix} 236 & 437 \\ 220 & 685 \end{pmatrix} \mu K \cdot \sqrt{s}$$

$$\frac{16}{R16_D} = 3.3 \qquad \frac{NET_D}{3.3} = \begin{pmatrix} 242 \\ 371 \end{pmatrix} \mu K \cdot \sqrt{s} \qquad \frac{16}{R16_{me}} = 2.5 \qquad \frac{NET_{me}}{2.5} = \begin{pmatrix} 160 & 297 \\ 149 & 466 \end{pmatrix} \mu K \cdot \sqrt{s}$$

Desert's sheet for 4 pixels (FWHM beam):

$$NET_{ds} := \begin{pmatrix} 1164 \\ 3322 \end{pmatrix} \cdot \mu K \cdot \sqrt{s}$$

Desert calculates $NET_{ds}=NET_{1pixds}/2$ as expected for 4 pixels and a poissonian NEP. Though he neglects bunching noise his scalling is close to mine because his hypothesis on pixel size and transmissions give by cahnce $4/R4_D=1.9$. Hence the problems come again from EffLP in P_{rj} and P_{sup} in NEP, indeed:

$$\frac{\sqrt{4} \cdot NEP_{ds_{ii}} \cdot T_{RJt}}{4\sqrt{2} \cdot EffLP_{ii} \cdot P_{rjD_{ii}}} = \begin{pmatrix} 1160 \\ 3253 \end{pmatrix} \mu K \cdot \sqrt{s}$$

as for 1 pixel: $\frac{NET_{ds_{ii}}}{NET_{ND_{ii}}} = \begin{pmatrix} 2.7 \\ 5.1 \end{pmatrix}$

Small difference with his sheet again due to the 3rd order approx of the band integral (whereas I use 1st order).

My results from III.5.:

0.5Fλ pixel

$$NET_{m_{ii,v}} := pNET_{Tb_{ii+1,v}}$$

$$\frac{NET_m}{\sqrt{2}} = \begin{pmatrix} 419 & 738 \\ 391 & 1165 \end{pmatrix} \mu K \cdot \sqrt{s}$$

1Fλ pixel (FWHM beam)

$$NET_{1F\lambda_{ii,v}} := pNET_{Tb1_{ii+1,v}}$$

$$\frac{NET_{1F\lambda}}{\sqrt{2}} = \begin{pmatrix} 255 & 474 \\ 233 & 742 \end{pmatrix} \mu K \cdot \sqrt{s}$$

=> my approximate calculations are compatible with the exact calculs from III.5..

Remark: the definition of NET used here is between the "technical" pNET and the "practical" bNET from III.5. Indeed the sampling factor ($\sqrt{2}$) is included here, but not the observing mode (η_o).

<u>Noise Equivalent Flux Density (NEFD):</u>

General formula compatible with Desert's formulation (F_{pt} = flux from a point source, P_{bpt} = power in the main beam):

$$NEFD := \eta_o \cdot \frac{NEP_b \cdot F_{pt}}{\sqrt{2} \cdot P_{bpt}}$$

The index b (in NEP_b and P_b) referes to the beam and means this definition does not depend on the pixel size.
The factor η_o is the observing mode efficiency; using Desert's notation:

$$\eta_o := \sqrt{\frac{2}{EffMod}}$$

η_o allows the definition of a NEFD directly proportional to integration time. It counts for the signal modulation used to suppress background, and introduces 2 terms: (1) the fraction of time actually spent on the source (typically 80% for On-The-Fly observing mode, and 45% for On-Off [Desert]), the remaining being mainly spent on the background reference, (2) the doubling of the background noise introduced with the subtraction. On-Off is often used with horns, but for filled arrays OTF seems better [IRAM bolo meeting 2008, GISMO].

$$\eta_o := \sqrt{\frac{2}{0.8}}$$

$$\eta_o = 1.58$$

1st order approximation using quantities calculated previously:

(η_{beam} = integral of the diffraction patern up to the standard size: η_{HP} for the FWHM or η_{Beff} for the full main beam)

$$NEFD := \frac{\eta_o \cdot RN \cdot NEP \cdot po}{\sqrt{2} \cdot \eta_{beam} \cdot A \cdot tsky \cdot \Delta\nu}$$

Link to the NET defined above using 1st order approximation: $\mathrm{NEFD} := \eta_o \cdot \mathrm{RN} \cdot \frac{F_{eff}}{\eta_{beam}} \cdot \frac{\mathrm{NET} \cdot 2 \cdot k \cdot \Omega_b}{\lambda^2}$

As previously, the standard size chosen is the beam FWHM (1Fλ), so 4 pixels coadded.

Problem: in his sheet Desert calculate P_{pt} for one pixel and for the full main beam. Since the ratio for his scalling is similar to mine, let use also my ratio for the scalling from one pixel to the beam FWHM:

$$\frac{\eta_{HPme_{ii}}}{\eta_{pixme_{ii}}} = \begin{pmatrix} 3.0 \\ 3.0 \end{pmatrix}$$

$$\mathrm{NEFD}_{D_{ii}} := \frac{\eta_o \cdot \mathrm{NEP}_{ND_{ii}} \cdot F_t}{\sqrt{2} \cdot 3 \cdot P_{ptD_{ii}}}$$

$$\boxed{\mathrm{NEFD}_D = \begin{pmatrix} 5.7 \\ 17.5 \end{pmatrix} \mathrm{mJy} \cdot \sqrt{s}}$$

$$\mathrm{NEFD}_{me_{ii,v}} := \frac{\eta_o \cdot \mathrm{NEP}_{Nme_{ii,v}} \cdot F_t}{\sqrt{2} \cdot P_{ptme_{ii,v}}} \cdot \frac{\eta_{pixme_{ii}}}{\eta_{HPme_{ii}}}$$

$$\boxed{\mathrm{NEFD}_{me} = \begin{pmatrix} 2.5 & 4.8 \\ 3.2 & 10.7 \end{pmatrix} \mathrm{mJy} \cdot \sqrt{s}}$$

In his sheet, Desert does several calculations of NEFD:

- 1st calculation: he uses the formula with the FWHM beam NET and Ω_b (Glh in his notation).
- 2nd calculation: he uses the formula with the NEP of one pixel, 1/4th of the full beam P_{pt} and the inverse of the scalling needed to get the NEP of the FWHM !

The 2nd calculation looks wrong, but he finds for both:

$$\mathrm{NEFD}_{ds} := \begin{pmatrix} 6.3 \\ 17.8 \end{pmatrix} \cdot \mathrm{mJy} \cdot \sqrt{s}$$

My results from III.5. (for a 1Fλ pixel = 4 coadded 0.5Fλ pixels):

$$\mathrm{NEFD}_{m_{ii,v}} := \mathrm{bNEFD}_{Tb_{ii+1,v}}$$

$$\mathrm{NEFD}_m = \begin{pmatrix} 2.6 & 4.8 \\ 3.5 & 10.7 \end{pmatrix} \mathrm{mJy} \cdot \sqrt{s}$$

=> My approximate formulas give again results close to the exact calculations from III.5.

Redoing Desert's calculations with his formulas and his values:

FWHM throughput in his formula: $\mathrm{Glh} := \frac{4 \cdot S\Omega_D}{A}$ $\quad \mathrm{Glh} = \begin{pmatrix} 4.08 \\ 1.33 \end{pmatrix} 10^{-9} \mathrm{sr}$ as in his sheet

Desert uses On-Off for his "initial case", not OTF. This is not a mistake, just a choice of configuration. For the calculation of NEFD_D I prefer keeping OTF for an easier comparison with NEFD_{me}:

$$\eta_D := \sqrt{\frac{2}{0.45}} \qquad \eta_D = 2.11$$

$$\eta_D \cdot \mathrm{NET}_{ds_{ii}} \cdot \frac{2k\,\mathrm{Glh}_{ii}}{\left(\lambda_{D_{ii}}\right)^2} = \begin{pmatrix} 6.3 \\ 17.9 \end{pmatrix} \mathrm{mJy} \cdot \sqrt{s} \qquad \frac{\eta_D \cdot \mathrm{NEP}_{ds_{ii}} \cdot F_t}{\frac{5 \cdot P_{ptD_{ii}}}{4}} \cdot \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{4}} = \begin{pmatrix} 6.3 \\ 17.5 \end{pmatrix} \mathrm{mJy} \cdot \sqrt{s}$$

=> Same result as Desert's sheet. So I understand his calculations, but disagree with them:

Desert's 2nd calculation looks wrong to me because (1) P1mJy in Desert sheet = 5*P_{ptD} = power in the full beam, and 5*P_{ptD}/4 is NOT the power in the beam FWHM, (2) NEP_{ds} is the NEP in one pixel, so $\mathrm{NEP}_{ds}/\sqrt{4}$, is the NEP in a surface smaller than a pixel ! But by chance $4/\sqrt{4} = \sqrt{4}$ is the poissonian scalling to get the NEP of the beam FWHM from 1 pixel, and 5*P_{ptD} is the power in the FWHM if EffLP is the efficiency of the beam FWHM (η_{HP}), not of the full beam (η_{Beff}) as suggested by the formula of P_{ptD} (hence a ratio 3/5 with my method, see below) ! With this gymnastics both errors cancel each other and the 2nd calculation is NEFD for the beam FWHM. Now replacing NET, P_{pt} and P_{ri} with their parent formulas shows that the 1st and 2nd calculation are equivalent... but imply that EffLP is the pixel efficiency for extended sources in P_{rj} !

From previous observations, 4 factors should explain the difference between Desert's result and my calculation using his transmission factors: (1) the scalling from P_{pt} on one pixel to the power in the beam FWHM is a factor 3, not 5; (2) NEP_{ds} contains a wrong contribution of P_{sup} and should be replace by NEP_D; (3) Desert scalling of the NEP from a 0.5Fλ pixel to FWHM is purely poissonian whereas with bunching it should be $R4_D$; (4) Desert assumes OnOff, not OTF.

Thus: $NEFD_{D_{ii}} \cdot \frac{3}{5} \cdot \frac{NEP_{ds_{ii}}}{NEP_{D_{ii}}} \cdot \frac{2}{R4_D} \cdot \frac{\eta_D}{\eta_o} = \begin{pmatrix} 6.4 \\ 17.7 \end{pmatrix} mJy \cdot \sqrt{s}$

=> Same result as the calculations above. So the difference between me and Desert's formula is explained.

In his sheet, Desert does a <u>3rd calculation</u> for the NEFD, using the NET like the 1st calculation, but this time it is the NET of one 0.5Fλ pixel and the corresponding solid angle, and he introduces a scalling factor 1.7 that he calls "*intégrale pondérée optimalement de mesure de flux d'une source ponctuelle par rapport au bruit d'un pixel*". Though this sentence is quite criptic, it looks like this factor is the right scalling to calculate the power from a point source in the FWHM from the power in the full beam.

Indeed Desert says 1/5th of the total power is in the central pixel (see P_{pt}) and I showed above that the ratio of the integral of the beam to 1Fλ versus the integral to 0.5Fλ is a factor 3:

Thus pixel to Full beam vs pixel to FWHM: $\frac{5}{3} = 1.7$

including effects of surface errors I find: $\frac{\eta_{Beffme_{ii}}}{\eta_{HPme_{ii}}} = \begin{pmatrix} 1.6 \\ 1.6 \end{pmatrix}$

So if my interpretation of Desert's 1.7 factor is correct the difference between his result and mine should be exlained with the following formula (attention! don't forget that downscalling $NEFD_D$ from 4 to 1 pixels needs a division by $R4_D$):

$\frac{NEFD_{D_{ii}}}{R4_D} \cdot \frac{NEP_{ds_{ii}}}{NEP_{D_{ii}}} \cdot \frac{\eta_D}{\eta_o} = \begin{pmatrix} 5.3 \\ 14.8 \end{pmatrix} mJy \cdot \sqrt{s}$

Which is close to his value: $NEFD_{ds} := \begin{pmatrix} 5.3 \\ 15.2 \end{pmatrix} \cdot mJy \cdot \sqrt{s}$

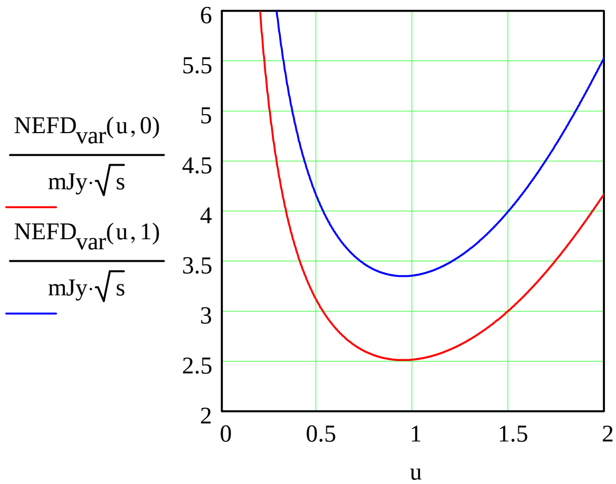
Verification redoing the calculation with his values and formula (attention! $2NET_{ds}$ is the NET for one pixel):

$1.7 \cdot \eta_D \cdot \left(2 \cdot NET_{ds_{ii}}\right) \cdot \frac{2 \cdot k \cdot S\Omega_{D_{ii}}}{A \cdot \left(\lambda_{D_{ii}}\right)^2} = \begin{pmatrix} 5.3 \\ 15.2 \end{pmatrix} mJy \cdot \sqrt{s}$

=> From the comparison of Desert's 1st and 2nd calculations with my calculation of $NEFD_D$, the factors (2), (3), and (4) remain. The small additional difference is due to the 3rd order approximation of the band integration (see NET).
It may seem strange that Desert uses the factor $1.7=\eta_{Beff}/\eta_{HP}$ (full beam to half power efficiency) whereas the rest of the formula uses terms calculated for a single pixel ! This oddity is explained remembering that Desert's deffinition of NET included the $EffLP=\eta_{Beff}$ in his calculation of Prj, extracting this factor from his formula, then only remains $1/\eta_{HP}$ as expected from the theoretical formula linking NEFD to NET shown above. Thus Desert's 1st and 3rd calculations inherit from the problem of EffLP in NET, itself inherited from the calculation of Prj, but the problem is corrected thanks to the odd scalling factor η_{Beff}/η_{HP} !

Now let's go back to my calculation with my transmission parameters and have a look at the evolution of the NEFD with the diameter u of the disc of integration of the beam (u=0.5 <=> 1*0.5Fλ pixel, u=1 <=> 4*0.5Fλ pixels coadded, u=2 <=> ~16*0.5Fλ pixels coadded, etc.). For the optimal atmospheric conditions one gets:

$$NEFD_{var}(u,ii) := \frac{\eta_o \cdot NEP_{me_{ii,0}} \cdot F_t}{\sqrt{2} \cdot P_{ptme_{ii,0}}} \cdot \sqrt{\frac{1}{2} \cdot \left[(2\cdot u)^2 + (2\cdot u)^3\right]} \cdot \frac{\eta_{pixme_{ii}}}{\eta_b\left(u, \nu_{ii+1}\right)}$$



Verification with the results from III.5. for u = 0.5, 1 and 2:

$$NEFD_{pm_{ii,v}} := \frac{\eta_o}{\sqrt{2}} \cdot pNEFD_{Tb_{ii+1,v}}$$

u=0.5: $NEFD_{pm} = \begin{pmatrix} 3.2 & 5.6 \\ 4.4 & 12.7 \end{pmatrix} mJy\cdot\sqrt{s}$

u=1: $NEFD_{m} = \begin{pmatrix} 2.6 & 4.8 \\ 3.5 & 10.7 \end{pmatrix} mJy\cdot\sqrt{s}$

$$NEFD_{fbm_{ii,v}} := \frac{\eta_o}{\sqrt{2}} \cdot pNEFD_{Tb2_{ii+1,v}}$$

u=2: $NEFD_{fbm} = \begin{pmatrix} 3.9 & 7.3 \\ 4.9 & 16.0 \end{pmatrix} mJy\cdot\sqrt{s}$

=> This scalable formula of the NEFD is compatible with the exact calculations from III.5. Attention! it is correct only in the same range as the for the approximation Δ~1/u in the bunching noise, that is to say in the range 0.5<u<4 (see III.4.). The behaviour of the curves shows that as expected the optimal point source sensitivity is reached for the beam FWHM, thus the relevance to use this size as the standard size independent from a given pixel size.

Remark: the calculations of the NEFD shown here includes the observing mode factor η_o, whereas the calculations of the NET didn't. I made this choice to be compatible with the approach of Desert's sheet, but it seems incoherent to me: why only one of both sensitivity parameters should includes information on the observing mode ?

CONCLUSION OF THE COMPARISON BETWEEN F.X.DESERT CALCULATIONS AND MINE FOR THE SPECIFICATIONS OF AN OPTIMAL FILLED ARRAY AT THE 30M TELESCOPE

Though several significative differences exists between Desert's calculations and mine, the estimated optimal performances are consistent in less than an order of magnitude, with point source half power beam width (HPBW or FWHM) optimal sensitivity of few $mJy\cdot\sqrt{s}$ for the 1 and 2 mm wavelength bands and On The Fly observing mode. The increase of background due to bad weather degrades this sensitivity by a factor up to 5, but the sky noise inevitably present with bad weather is neglected here while its effect can be much worse. Nevertheless only best observing conditions and maximum dynamics deduced from the power received from strong sources are necessary to define the optimal pixel.

Although Desert's results and mine are more convergent than previous works, some differences between us are still too important to call it compatibility. There's mainly 2 types of disagreements: (1) hypothesis on the physical properties of some elements of the system, and (2) the formulation of some equations. There won't be a final version of this work before the most significative differences recapitulated below are addressed:

1) Opacity of the atmosphere: Desert choose a rather mediocre atmosphere, whereas I choose a good atmosphere for the definition of the required optimal pixel noise performances (I also show the effect of bad weather mainly for information, but also to give the goal of the desired dynamics).
2) Transmission of the filters: Desert choose rather bad filters (tf=15%), I choose good filters based on Cardiff's specifications (tf=66% with 7 filters in series). The non-transmitted part of my filters is totally converted in emissivity, whereas part of it could be reflected back to the sky, hence decreasing a bit the NEP and NEFD, so increasing the constraint on the pixel performance, and the difference between Desert and I ! Desert filters are implicitly supposed highly reflective, otherwise their contribution would dominates the background.
3) If EffLP = B_{eff} as suggested by its use in most calculations, then its value for the 1mm band is abnormally low. Still, I don't have a final answer on this subject because I haven't fixed yet the problem I discovered comparing B_{eff} measurements with heterodyne feedhorns and deductions from antenna tolerance theory and holography measurement of the antenna surface errors.
4) Desert uses EffLP=B_{eff} in P1KRJ, whereas I think F_{eff} sould be used instead.
5) Desert doesn't use the same esup and tfilt in his calculation of Psup than in his calculation of NEPsup, which not calculated from Psup, but from a 3rd order approximation of the brightness B(Tsup) for a better approximation of the integral in the bandwidth !
6) Desert's bunching NEP (boson part) is polarized (introduction of a factor $\sqrt{2}$) whereas that is not the case for the other components.
7) Desert's NET inherits the problems from P1KRJ and NEPsup.
8) Desert's NEFD 1st calculation inherits the same problems as NET, in particular the efficiency hidden in his P1KRJ implies his result is NOT the NEFD for the beam FWHM.
9) Desert's NEFD 2nd calculation inherit the problem of NEPsup and uses weird scalling ratios for NEP and Ppt to find a formula equivalent to his 1st calculation.
10) The results of Desert's 3rd calculation is correct in terms of size scalling to get NEFD of the beam FWHM, but this is accomplished thanks to a "pickpoket trick": he applies the scalling factor from full beam efficiency to FWHM efficiency to single pixel quantities, so that the beam efficiency hidden in the P1KRJ of his NET is compensated correctly ! Thus in the end only remains the problem of Psup in his result.
11) Why including the observing mode factor in NEFD but not in NET ? I think it would be more fair to use the same factors for both sensitivity parameters, isnt'it ?

Last important remark: If the pixel efficiency is not included in the definition of the NEFD (as eNEFD in III.5.) this can be highly misleading in terms of sensitivity interpretation (eNEFD keeps decreasing with pixel size). Hence I agree that NEFD should always refers to point source detection and always includes an efficiency term related to the diffraction pattern (so never use eNEFD). I also agree that giving the sensitivity in a unit allowing a direct calculation of the detection time at a standard resolution is very useful for the observer, but such NEFD contains several free parameters that must be specified by the authors: (1) polarization factor, (2) size of the image portion used for the detection compared to the diffraction pattern, and (3) hypothesis on the observing mode. Without these indications, values are always ambiguous and confusing. This remark stands also for the NET...

All the calculations from this document indicate that compared to MAMBO 2 a factor ~10 improvement in sensitivity could be reached at the 30m telescope.