III.8) Pixel architecture comparison in terms of maps sizes.

Both chapters III.6. and III.7 showed that the number of pixels and filling factors are important criterias of the instrument performances. One can define various parameters to include them into one calculation. Below are two possible choices offering the advantage to be very eloquent in terms of practical instrument performance. Both use solid angles.

Introduction

Reminder: wavelength (λ), projection in the sky of the pixels physical size (Θ=u*λ/D), and angular sizes of the FWHM beams (θ=k_g*λ/D) they intercept:

$$\lambda = \begin{pmatrix} 3.20 \\ 2.05 \\ 1.25 \\ 0.87 \end{pmatrix} \mathrm{mm} \quad \Theta_b(\lambda) = \begin{pmatrix} 10 \\ 7 \\ 4 \\ 3 \end{pmatrix} \mathrm{as} \quad \Theta_h(\lambda) = \begin{pmatrix} 44 \\ 28 \\ 17 \\ 12 \end{pmatrix} \mathrm{as} \quad \theta_b(\lambda) = \begin{pmatrix} 23 \\ 15 \\ 9 \\ 6 \end{pmatrix} \mathrm{as} \quad \theta_h(\lambda) = \begin{pmatrix} 26 \\ 16 \\ 10 \\ 7 \end{pmatrix} \mathrm{as}$$

FOV stutied and number of pixels in compact arrays, using a square paving for bare pixels (N_b=(π/4)*(fov/Θ_b)^2), and a hexagonal paving for feedhorns (N_h=(π/(2*3$^{1/2}$))*(fov/Θ_h)^2):

$$\mathrm{fov} = (3.5 \quad 4.8 \quad 7.4 \quad 10.0)\ \mathrm{am}$$

$$N_b = \begin{pmatrix} 286 & 538 & 1279 & 2336 \\ 697 & 1312 & 3117 & 5693 \\ 1876 & 3528 & 8385 & 15312 \\ 3872 & 7283 & 17309 & 31609 \end{pmatrix} \qquad N_h = \begin{pmatrix} 21 & 39 & 92 & 169 \\ 50 & 95 & 225 & 411 \\ 135 & 255 & 605 & 1105 \\ 279 & 526 & 1249 & 2281 \end{pmatrix}$$

R.Zylka style for fast calculations [private email]: filling with 2θ_h spacing, so for a 10 arcmin FOV and 11 arcsec pixels (1.2mm):

$$N_{hZ} := \frac{\pi}{4}\cdot\frac{2}{\sqrt{3}}\cdot\left(\frac{10\cdot \mathrm{am}}{22\mathrm{as}}\right)^2 \qquad N_{hZ} = 675 \qquad N_{hZ} := 673$$

Zylka's pixels size and gaps are the same as MAMBO

Zylka/MAMBO style vs my optimal filling: $\frac{1105}{673} = 1.64$

Remark: MAMBO pixels sample full beams (not Nyquist !), so 1.2x bigger than HPBW: $k_g(T_{eh})\cdot\frac{1.22}{1.03}\cdot\frac{\lambda_M}{D} = 11\ \mathrm{as}$ (taper: T_{eh}=10dB)

Effective solid angle: $\Omega e_x := \int_0^{\Omega_x} I_n\, d\Omega$ r=θ*πD/2λ

Gaussian beam approximation: $\sigma_g(eT) := \frac{k_g(eT)\cdot\pi}{2\sqrt{2\ln(2)}}$ $G_g(r,eT) := \exp\left(\frac{-r^2}{2\cdot\sigma_g(eT)^2}\right)$

from σ, $\theta_{eff} = \theta_{fwhm}/(\ln(2))^{1/2}$ => $\Omega e = (\pi/4)*\theta_{eff}^2$ $\frac{\pi}{4\cdot\ln(2)} = 1.13309$

Total receiver source angle (Ω_N=N*Ω_{pix}):

Verification:

$$\Omega_{Nb_{0,j}} := \frac{\pi}{4}\cdot\left(\mathrm{fr}\cdot\mathrm{fov}_{0,j}\right)^2 \qquad \boxed{\Omega_{Nb} = (3.1 \quad 5.9 \quad 14.0 \quad 25.5)\ 10^4\cdot\mathrm{as}^2}$$

$$N_{b_{0,j}}\cdot\left(\Theta_b(\lambda_0)\right)^2 = \begin{pmatrix} 3.1 \\ 5.9 \\ 14.0 \\ 25.5 \end{pmatrix} 10^4\cdot\mathrm{as}^2$$

$$\Omega_{Nh_{0,j}} := \frac{\pi}{2\cdot\sqrt{3}}\cdot\frac{\pi}{4\cdot\ln(2)}\cdot\left(\mathrm{fov}_{0,j}\cdot\frac{k_g(T_{eh})}{u_h}\right)^2$$

$$\boxed{\Omega_{Nh} = (1.5\ \ 2.9\ \ 6.9\ \ 12.6)\,10^4\cdot as^2}$$

$$\frac{N_{h_{0,j}}\cdot\pi}{4\cdot\ln(2)}\cdot\left(\theta_h(\lambda_0)\right)^2 = \begin{pmatrix}1.5\\2.9\\6.9\\12.6\end{pmatrix}10^4\cdot as^2$$

Zylka's style: filled array: $\frac{\pi}{4}\cdot(10am)^2 = 28.3\,10^4\cdot as^2$

horns: $N_{hZ}\cdot 1.13309\cdot(11as)^2 = 9.2\,10^4\cdot as^2$

Zylka's argument for quick calculations: instruments source angle ratio ~= maping speed ratio: $\frac{\Omega_{Nb_{0,0}}}{\Omega_{Nh_{0,0}}} = 2.02$

Zylka's choice to use $2\theta_h$ (vs Θ_h for me) spacing between horns and none (vs fr for me) between bare pixels gives: $\frac{\pi\cdot(5am)^2}{9.2\cdot 10^4 as^2} = 3.07$

From III.7: with shot noise only, same filters and without the F_{eff} subetelty, the extended source speed ratio = filling factor (N_b/N_h=13.9) * pixel throughput ratio ($\upsilon_b/\upsilon_h=\Omega_b/\Omega_h$) = Ω_{Nb}/Ω_{Nh}. But this calculation gives: $13.9\cdot\frac{\upsilon_b}{\upsilon_h} = 2.71$

<u>==> Which is right, and what is the link between Zylka's & Griffin's methods ?</u>
Reminder: using a slightly different horn shape hypothesis, Griffin has a smaller υ_h, and find a speed ratio =2.9 totally compatible with my method from III.7.

Reminders: From II.1.: $A\Omega_s(\upsilon,\lambda) := \upsilon\cdot\lambda^2$ $\quad \upsilon_{bare}(U_p) := \frac{\pi}{4}\cdot U_p$ $\quad \upsilon_{horn}(u) := \varepsilon_{sg}(T_e(u))$

From I.3.: $\varepsilon_{sg}(eT) := \left(1 - \exp\left(\frac{-1}{2\,\sigma_t(eT)^2}\right)\right)$

but ATTENTION ! σ_t is the width factor of the taper function projected on antenna, NOT the beamwidth @ the horn (σ_g) !

$$\sigma_t(eT) := \sqrt{\frac{10}{2\cdot eT\cdot\ln(10)}} \qquad \sigma_t(T_{eh}) = 0.5 \qquad \sigma_g(eT) := \frac{k_g(eT)\cdot\pi}{2\sqrt{2\ln(2)}} \qquad \sigma_g(T_{eh}) = 1.6$$

Decomposing the receiver source angle ratio Ω_{Nb}/Ω_{Nh} in filling factor and individual pixel effective surface in units of λ/D (so pixel sizes in terms of beam solid angle in the sky, not angular response through instrument pupil !) one gets:

$$13.9\cdot\frac{(fr\cdot u_b)^2}{\frac{\pi}{4\cdot\ln(2)}\cdot k_g(T_{eh})^2} = 2.02 \qquad (fr\cdot u_b)^2 = 0.23 \qquad \frac{\pi}{4\cdot\ln(2)}\cdot k_g(T_{eh})^2 = 1.55$$

using width of gaussian: $\frac{2}{\pi}\cdot\sigma_g(T_{eh})^2 = 1.55$

Now decomposing the throughput efficiencies (portion of sky it really seen by pixels = sizes corrected by the angular response through the instrument pupil, i.e. by the spill-over):

$$13.9\cdot\frac{\frac{\pi}{4}\cdot(fr\cdot u_b)^2}{\varepsilon_{sg}(T_{eh})} = 2.71 \qquad \frac{\pi}{4}\cdot(fr\cdot u_b)^2 = 0.18 \qquad \varepsilon_{sg}(T_{eh}) = 0.91 \qquad k_g(T_{eh})^2 = 1.4$$

Using the same scalling from pixel effective surface to throughput in units of λ^2 (= throughput efficiency) as for bare pixel, one gets for the horn gaussian beam approximation:

$$A\Omega_{eg}/\lambda^2 = \frac{\pi}{4}\cdot\left(\frac{\pi}{4\cdot\ln(2)}\cdot k_g(T_{eh})^2\right) = 1.22$$

not= 1 , so the horn is not single-moded ?!

Remark: as expected for the gaussian beam approximation, the same factor as in Downes calculation of B_{eff} appears:

$$\frac{\pi^2}{16 \cdot \ln(2)} = 0.8899$$

==> the problem comes from the gaussian approximation overestimating the size of the beam so that the throughput is not single-moded anymore, which is also obtained with $1/\varepsilon_{Gg}$ (see I.3.) !

In another hand the calculation of the gaussian tappered beam throughput using $1/\varepsilon_{tg}$ gives 1.12 (see I.3.), not 1 !!!

=> does the truth lies between the 2 results above ?

$$\frac{1}{\varepsilon_{Gg}(T_{eh})} = 1.22$$

$$\frac{1}{\varepsilon_{tg}(T_{eh})} = 1.12$$

==> Zylka's method is fast but uses approximations overestimating the actual horns throughput: (1) the spill-over is neglected, (2) the taper efficiency is taken into account in the calculation of the effective solid angle (through the factor $\pi/4\ln(2)$), but it uses the gaussian approximation of the beam giving a throughput bigger than expected for a single-moded feedhorn ! Nevertheless this overestimation of horn throughput is smaller than the effect of the non optimal $2\theta_h$ inter pixel gap instead of the more compact $2\Theta_h$ so that in the end his calculation of bare pixel vs horn receiver source angles gives 3.07 instead of 2.02, whereas using the single moded throughput and optimal filling as Griffin and I in III.7 gives 2.71 !!

ATTENTION: both Zylka's and Griffin methods assume implicitly that the Signal to Noise is proportional to the detector effective size; wich implies the noise must be poissonian. So they are valid only when the shot noise dominates (bunching noise negligible, which may not be true !).

Zylka's suggestion for an alternative reformulation of the instrumental comparison:

FOV ratio (rfov) of a filled array with the same integrating angle as a horn array.

For a pixel architecture p: $\Omega_{Np}=a_p * fov_p^2$, where a is a multiplicative factor. For similar fov the speed ratio is $sre_{bh} = \Omega_{Nb}(fov)/\Omega_{Nh}(fov) = a_b/a_h$ ==> $fov_b = fov_h/(sre_{bh})^{1/2}$.

Using Zylka's example: $rfov_Z := \frac{1}{\sqrt{3.07}}$ $\quad rfov_Z = 0.6$ $\quad$ => fov of a filled array equivalent to a 10 am horns: $10am \cdot rfov_Z = 5.7\,am$

Number of pixels: $N_{hZ} = 673$ $\quad$ 11 as horns for 10 am $\quad \frac{N_{hZ} \cdot 1.13309 \cdot (11 \cdot as)^2}{(0.5 \cdot 11as)^2} = 3050$ $\quad$ bare pixels Nyquist sampling a horn HPBW at λ=1.2mm on 5.7am fov.

Same argument using my speed ratios:

shot noise + buncing noise $\quad rfov_{e_{i,v}} := \frac{1}{\sqrt{sreN_{bh_{i,v}}}}$ $\quad rfov_e = \begin{pmatrix} 0.3 & 0.3 \\ 0.3 & 0.3 \\ 0.3 & 0.3 \\ 0.3 & 0.3 \end{pmatrix}$ $\quad 10am \cdot rfov_{e_{0,0}} = 3.1\,am$

Number of pixels: $N_{h_{2,3}} = 1105$ $\quad$ 2Fλ horns at 1.2mm λ on 10am fov $\quad \frac{\pi}{4} \cdot \left(\frac{3.1am}{\Theta_b(\lambda_2)}\right)^2 = 1630$ $\quad$ bare pixels sampling the untapered beam at 1.2mm λ on 3.1 am fov.

Same with Griffin, for shot noise only:

$rfov_G := \frac{1}{\sqrt{2.9}}$ $\quad 10am \cdot rfov_G = 5.9\,am$ $\quad \frac{\pi}{4} \cdot \left(\frac{5.9am}{\Theta_b(\lambda_2)}\right)^2 = 5906$ $\quad$ bare pixels sampling the untapered beam at 1.2mm λ on 5.9 am fov.

BUT THIS IS TRUE ONLY FOR EXTENDED SOURCES SINCE WE DON'T USE THE PIXEL EFFICIENCY !!! Extending this argument to point sources makes sense only for point source extraction from map, but is obviously totally meaningless for point source direct detection:

$$rfov_{e_{i,v}} := \frac{1}{\sqrt{spr\_map_{bh_{i,v}}}} \qquad rfov_e = \begin{pmatrix} 0.3 & 0.3 \\ 0.3 & 0.3 \\ 0.4 & 0.3 \\ 0.3 & 0.3 \end{pmatrix} \qquad 10am \cdot rfov_{e_{0,0}} = 3.3\,am \qquad \frac{\pi}{4} \cdot \left(\frac{3.3am}{\Theta_b(\lambda_2)} \right)^2 = 1848$$

Beside the mismatching numbers due to different hypothesis, there's one interesting conclusion of this reformulation: **instrument preformances are much more greedy for large FOVs when they use feedhorns than when they use filled arrays.**

Desert's suggestion for another eloquent instrumental comparison:

Solid angle mapped in the sky in 1 hour at a sensitivity of 1mJy at 1σ level with 1000 pixels (Ω_{1hmk}):

Time to detect a 1mJy source at 1σ

$$t_{ptb1_{i,v}} := \frac{1}{2} \cdot \left(\frac{NEP_{Tb_{i,v}}}{P_{ptob_{i,v}}} \right)^2 \qquad t_{pth1_{i,v}} := \frac{1}{2} \cdot \left(\frac{NEP_{Th_{i,v}}}{P_{ptoh_{i,v}}} \right)^2$$

Solid angle seen by a pixel: Same problem as before: should I use direct Ω, or corrected by throughput efficiency, or by pixel apperture efficiency (since mJy refers to point source) ?

$$\Omega_{br} := \Theta_b(\lambda)^2 \qquad \Omega_{hr} := \frac{\pi \cdot \theta_h(\lambda)^2}{4 \cdot \ln(2)}$$

? or ?

$$\Omega_{bt} := \frac{\pi}{4} \cdot \Theta_b(\lambda)^2 \qquad \Omega_{ht} := \varepsilon_{sg}(T_{eh}) \cdot \frac{\lambda^2}{A}$$

$$\Omega_{br} = \begin{pmatrix} 109 \\ 45 \\ 17 \\ 8 \end{pmatrix} as^2 \qquad \Omega_{hr} = \begin{pmatrix} 750 \\ 308 \\ 114 \\ 55 \end{pmatrix} as^2 \qquad \Omega_{bt} = \begin{pmatrix} 86 \\ 35 \\ 13 \\ 6 \end{pmatrix} as^2 \qquad \Omega_{ht} = \begin{pmatrix} 560 \\ 230 \\ 85 \\ 41 \end{pmatrix} as^2$$

Thus

$$\Omega_{1hmkb_{i,v}} := 1000 \cdot \Omega_{bt_i} \cdot \frac{1hr}{t_{ptb1_{i,v}}} \qquad \Omega_{1hmkh_{i,v}} := 1000 \cdot \Omega_{ht_i} \cdot \frac{1hr}{t_{pth1_{i,v}}}$$

$$\Omega_{1hmkb} = \begin{pmatrix} 10.0 & 6.6 \\ 2.4 & 0.8 \\ 0.5 & 5.6 \times 10^{-2} \\ 5.1 \times 10^{-3} & 3.9 \times 10^{-5} \end{pmatrix} deg^2 \qquad \Omega_{1hmkh} = \begin{pmatrix} 97.6 & 56.9 \\ 26.5 & 6.3 \\ 5.7 & 0.4 \\ 4.9 \times 10^{-2} & 2.6 \times 10^{-4} \end{pmatrix} deg^2$$

IS THIS CRITERIA REALLY USEFUL ?