The effect of the filter function in double-beam maps

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Date: 30.01.2001

Even in double-beam maps, the filter function needs to be considered: the telescope response $f(t)$ to the sky signal is convolved with the filter's time domain response $B(t)$, and the result is integrated over a time window of record length Δt, to yield some record $S(t_\mathrm{i})$:

$$S(t_\mathrm{i}) = \frac{1}{\Delta t} \cdot \int_{t_i}^{t_i+\Delta t} dt \int_{-\infty}^{t} f(t')B(t'-t)dt' \,. \tag{1}$$

Equation 1 is valid either for the on-phase or the off-phase of the signal. For a double-beam record of period Δt, we have

$$S_\mathrm{db}(t_\mathrm{i}) = \frac{1}{\Delta t} \cdot \int_{t_i}^{t_i+\frac{\Delta t}{2}} dt \int_{-\infty}^{t} f(t')B(t'-t)dt' - \frac{1}{\Delta t} \cdot \int_{t_i+\frac{\Delta t}{2}}^{t_i+\Delta t} dt \int_{-\infty}^{t} f(t')B(t'-t)dt' \,. \tag{2}$$

The above expression, after partial integration, may be rewritten as

$$\begin{aligned} S_\mathrm{db}(t_\mathrm{i}) = \quad & \textstyle\int_{t_\mathrm{i}}^{t_\mathrm{i}+\Delta t} \left(2\Theta(t - t_\mathrm{i} - \frac{\Delta t}{2}) - 1\right) t f(t) B(0) dt \\ & - \left[\left(2\Theta(t - t_\mathrm{i} - \frac{\Delta t}{2}) - 1\right) \textstyle\int_{-\infty}^{t} t f(t') B(t - t') dt'\right]_{t_\mathrm{i}}^{t_\mathrm{i}+\Delta t} \,, \end{aligned} \tag{3}$$

where $\Theta(t)$ is the step function. The first term in the above expression contains the filter function merely as a constant factor, that would cancel out in the calibration. The second factor does not cancel out when we calibrate, and its importance with the respect to the first one determines the error in a calibration that does not take into account the filter function [1].

To illustrate the effect, we demonstrate the case of a simulated map of Mars, with an assumed diameter of $9''\!.2$, and a pillbox-like surface profile. For assumed declination $\delta = 0°$, and hour angle 0^h, a map size of $120'' \times 120''$, a wobbler throw of $46''$, the on- and off-phase were simulated with NIC commands FIELD and SIMULATE. Only one central channel was considered. The obtained time series of both on- and off phase (wobbler period is 0.5 s) were resampled on finer timesteps (of 12.5 ms), to allow for a more accurate convolution with the filter function. The result (time series convoled with time-domain response of filter function, Figure) was re-binned to samples of 0.25 s length, spaced by 0.5 s (the remaining 0.25 s of the period were used for the other phase). The

[1] The frequency dependence of the filter function was kindly communicated by E. Kreysa, MPIfR.

simulated source image (without filter function) is slightly larger than the model image, convolved with a $11''$ beam, due to the beam smearing on the sky. The filter function is clearly visible in the deformation of the source, that should have (in the ideal case) an elliptic shape (see Figure 3). In the observations (Figure), the asymmetry due to the filter function is also clearly visible, though weaker than in the simulation. It is possible to artificially remove the image deformation by a positive ZIGZAG correction. The proper method, however, is to deconvolve **all** data from the filter response. Although the simulation shown here may be numerically still somewhat inaccurate (i.e. the discrete nature of the convolution), I do not expect the qualitative results to change significantly.

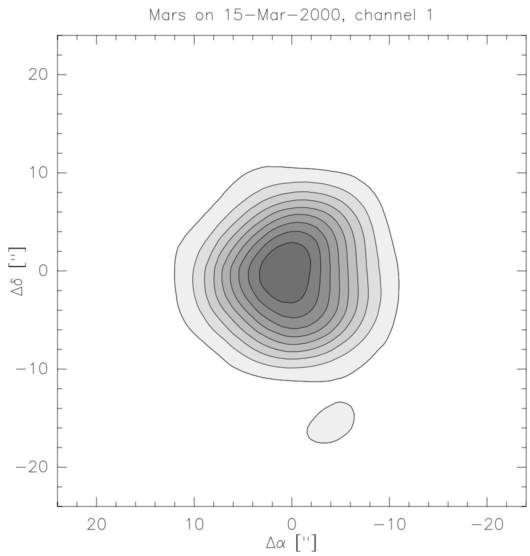

Figure 1: Mars, observed on Mars 15, 2000. Contour levels are at 10 to 90 % of peak flux, in steps of 10 %. The map was derived from channel 1 data only.

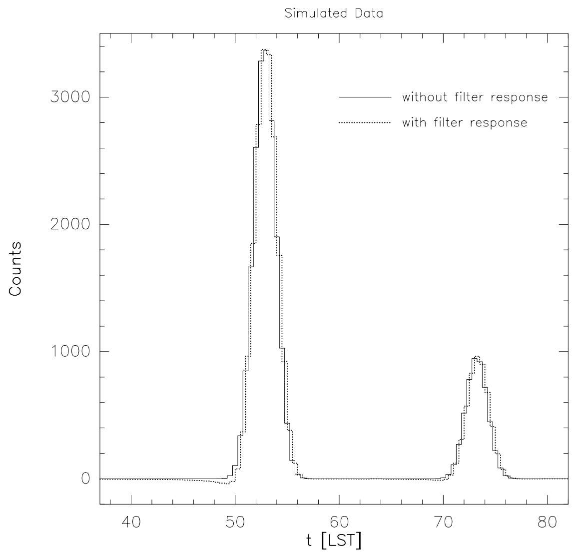

Figure 2: Time series of simulated data stream, for central channel, on-phase, without filter response (solid line) and with filter response (dashed line).

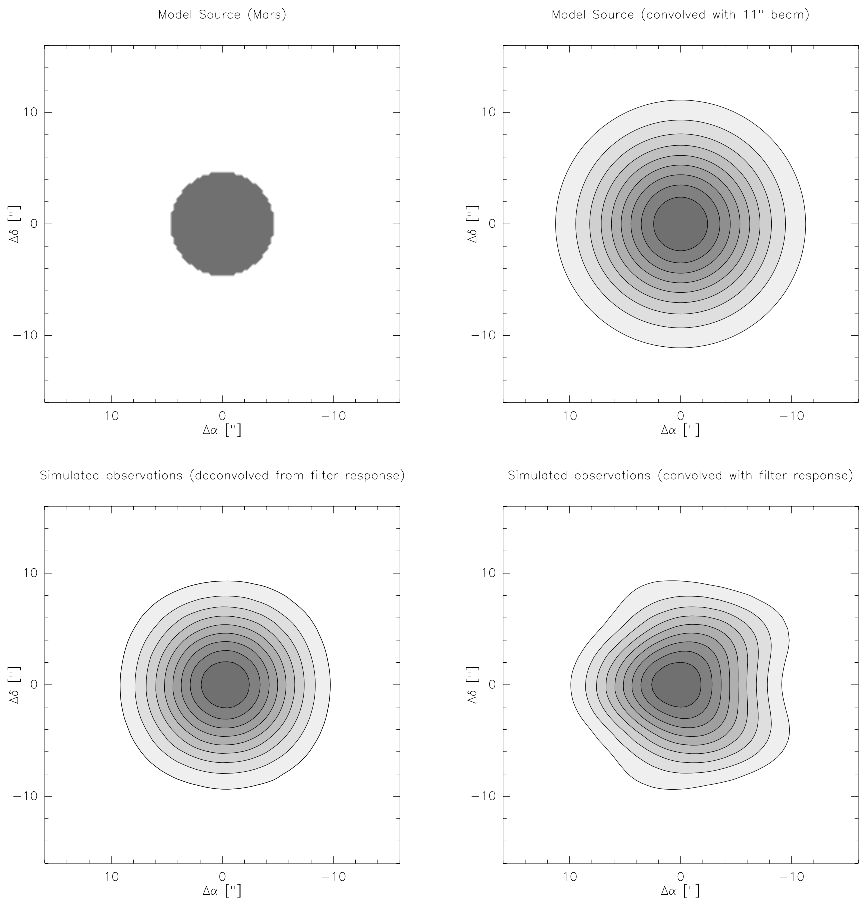

Figure 3: Effect of the time-domain response of the filter function in standard double-beam observations. All counts are normalized to unity, contours are at 10 % to 90 %, in steps of 10 %.