The response of the bolometer filter function

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1 The filter characteristics in the time domain

The filter function (Fig. 1) of the bolometer is that of a biquad filter, and is given by (E. Kreysa, priv. comm.)

$$\begin{aligned} b_\nu &= \frac{\alpha}{\left(\alpha - i\frac{\beta}{\nu}\right)(1+i\gamma\nu)} \\ &= \frac{\alpha(\alpha+\beta\gamma)}{(\alpha+\beta\gamma)^2+(\alpha\gamma\nu-\frac{\beta}{\nu})^2} - i\frac{\alpha\left(\alpha\gamma\nu-\frac{\beta}{\nu}\right)}{(\alpha+\beta\gamma)^2+(\alpha\gamma\nu-\frac{\beta}{\nu})^2}\,. \end{aligned} \tag{1}$$

The parameters α, β and γ are summarized in Tab. 1. The filter response in the time domain, $B(t)$, is given by its Fourier transform:

$$\begin{aligned} B(t) &= \int_{-\infty}^{+\infty} \frac{\alpha(\alpha+\beta\gamma)}{(\alpha+\beta\gamma)^2+(\alpha\gamma\nu-\frac{\beta}{\nu})^2}\cos(2\pi\nu t)d\nu \\ &- \int_{-\infty}^{+\infty} \frac{\alpha(\alpha\gamma\nu-\frac{\beta}{\nu})}{(\alpha+\beta\gamma)^2+(\alpha\gamma\nu-\frac{\beta}{\nu})^2}\sin(2\pi\nu t)d\nu\,. \end{aligned} \tag{2}$$

As expected, $B(t)$ is a real function, due to the hermiticiy of the filter function. The integral may be evaluated by means of the residuum calculus. We set

$$B(t) = F_1(t) + F_2(t) \tag{3}$$

with

Table 1: Parameters of the filter function b_ν.

$\alpha =$	1040	$\nu_{1,3}$	$= \pm 0.153\,\mathrm{Hz}$
$\beta =$	$159.155\,\mathrm{Hz}$	$\nu_{2,4}$	$= \pm 156.3\,\mathrm{Hz}$
$\gamma =$	$6.39609\,\mathrm{ms}$		

$$F_1(t) = \int_{-\infty}^{\infty} \Re(b_\nu) \cos(2\pi\nu t) d\nu \tag{4}$$

and

$$F_2(t) = \int_{-\infty}^{\infty} \Im(b_\nu) \sin(2\pi\nu t) d\nu \,. \tag{5}$$

The Fourier transform of the cosine part reads

$$\begin{aligned} F_1(t) &= \frac{1}{2}\int_{-\infty}^{\infty} \frac{\alpha p \nu^2}{p^2\nu^2 + (q\nu^2 - \beta)^2} (\exp(+2\pi i\nu t) + \exp(-2\pi i\nu t)) d\nu \\ &= \pi i \sum_{j=1}^{2} \mathrm{Res}\left\{ \frac{\alpha p s^2}{p^2 s^2 + (q s^2 - \beta)^2} \exp(+2\pi i t s) \right\}_{s=i\nu_j} \qquad (6) \\ &\quad -\pi i \sum_{j=3}^{4} \mathrm{Res}\left\{ \frac{\alpha p s^2}{p^2 s^2 + (q s^2 - \beta)^2} \exp(-2\pi i t s) \right\}_{s=i\nu_j} , \qquad (7) \end{aligned}$$

with the substitutions $p = \alpha + \beta\gamma$, $q = \alpha\gamma$, and where $i\nu_{\mathrm{j}}$ are the poles of the integrand in the upper respectively lower half of the complex plane, namely

$$\begin{aligned} i\nu_{1,2} &= +\sqrt{\frac{\beta}{q} - \frac{p^2}{2q^2} \pm \sqrt{\frac{p^4}{4q^4} - \frac{\beta p^2}{q^3}}} , \\ i\nu_{3,4} &= -\sqrt{\frac{\beta}{q} - \frac{p^2}{2q^2} \pm \sqrt{\frac{p^4}{4q^4} - \frac{\beta p^2}{q^3}}} . \qquad (8) \end{aligned}$$

Both poles are on the imaginary axis. The location of the poles in the complex plane is shown in Fig. 2. Similarly, we find for the sine part:

$$\begin{aligned} F_2(t) &= \frac{1}{2i}\int_{-\infty}^{\infty} \frac{\alpha \nu (q\nu^2 - \beta)}{p^2\nu^2 + (q\nu^2 - \beta)^2} (\exp(+2\pi i\nu t) - \exp(-2\pi i\nu t)) d\nu \\ &= \pi \sum_{j=1}^{2} \mathrm{Res}\left\{ \frac{\alpha s (q s^2 - \beta)}{p^2 s^2 + (q s^2 - \beta)^2} \exp(+2\pi i t s) \right\}_{s=i\nu_j} \qquad (9) \\ &\quad +\pi \sum_{j=3}^{4} \mathrm{Res}\left\{ \frac{\alpha s (q s^2 - \beta)}{p^2 s^2 + (q s^2 - \beta)^2} \exp(-2\pi i t s) \right\}_{s=\nu_j} , \qquad (10) \end{aligned}$$

The response function $B(t) = F_1(t) + F_2(t)$ is shown in Fig. 3.

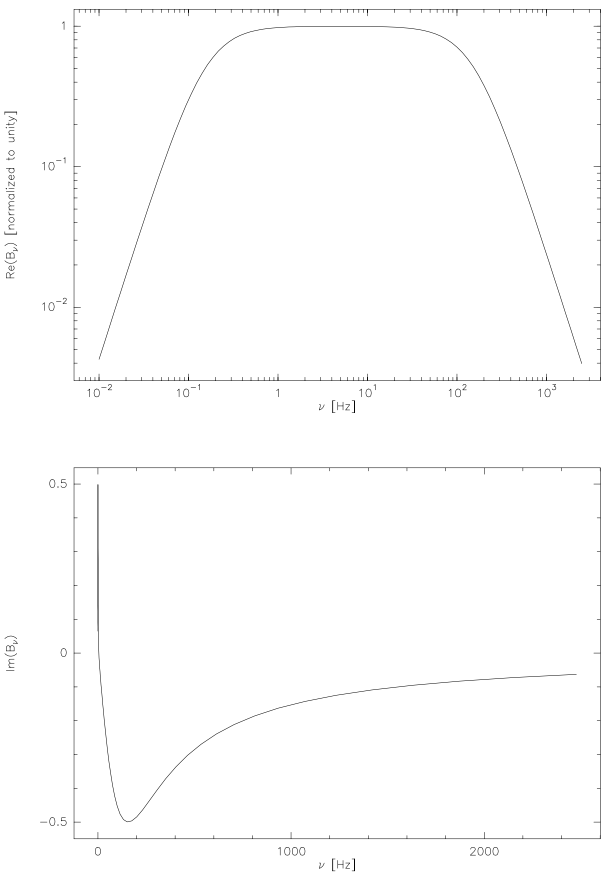

Figure 1: Real and imaginary part of the filter function $b(\nu)$ (eq. 1).

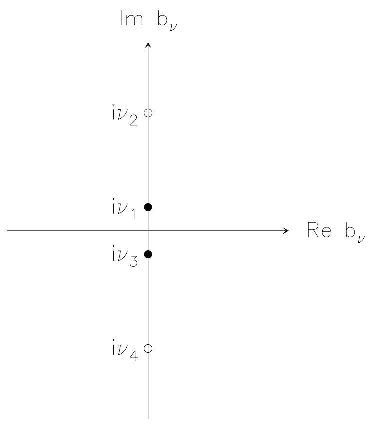

Figure 2: Location of the poles of the complex filter function in the complex plane (not to scale).

2 The filter response to the observations

In the observations, the telescope response $f(t)$ to the sky signal is convolved with the filter's time domain response, and the result is integrated over a time window of record length Δt:

$$S(t_\mathrm{i}) = \frac{1}{\Delta t} \cdot \int_{t_i}^{t_i+\Delta t} dt \int_{-\infty}^{t} f(t')B(t'-t)dt' \,. \tag{11}$$

An example of the time-domain response to a fast-scanned observation of Mars is shown in Figure 4. After deconvolution, the original signal may be recovered, i.e.

$$f(t) = \exp\left(-4\log{(2)}(t-t_0)^2 \cdot \left(\frac{V_\mathrm{AZ}}{\mathrm{FWHM}}\right)^2\right) , \tag{12}$$

where the signal width (in *arc sec*) is

$$\mathrm{FWHM} = \sqrt{\Theta^2_\mathrm{beam} + \Theta^2_\mathrm{source}} \,, \tag{13}$$

and where V_AZ is the scanning speed of the telescope (in *arc sec*/s).

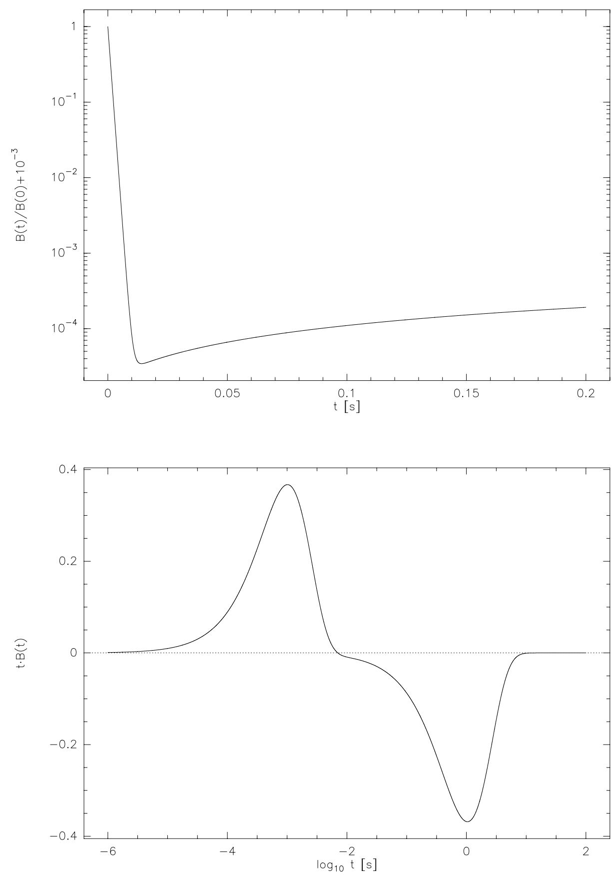

Figure 3: Time-domain response of the filter function (eq. 3). **Upper:** Note the offset introduced in the response to allow for the logarithmic plot. **Lower:** $t \cdot B(t)$ vs. $\log_{10}(t)$. The areas above and below the time axis are equal in size, because the filter amplitude falls down to zero at $\nu = 0$.

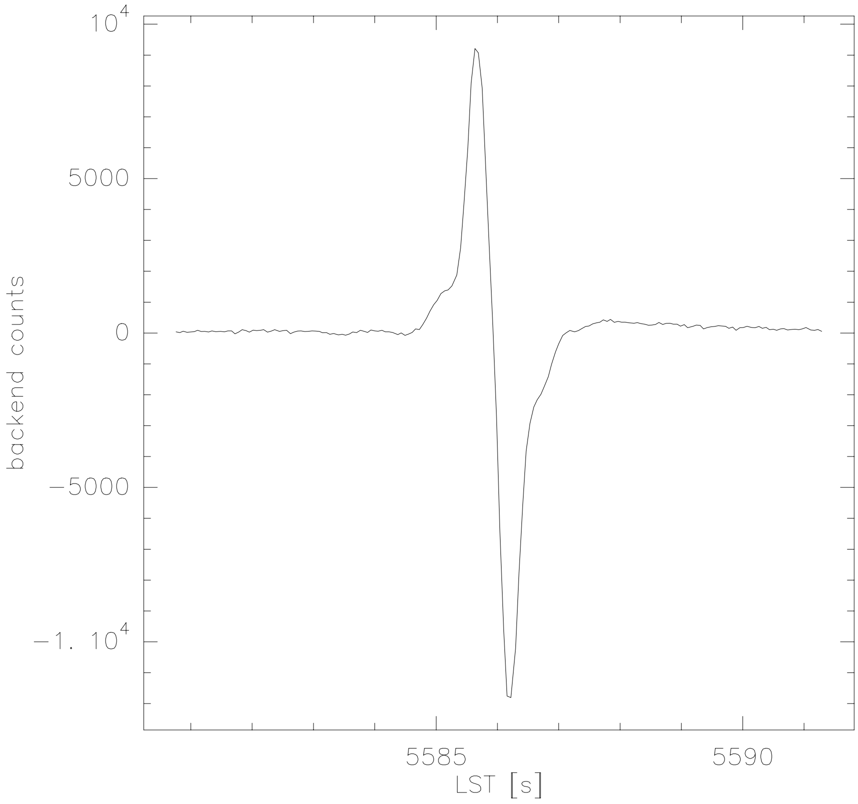

Figure 4: Subscan of a fast-scanned map of Mars.