Simulating correlated skynoise

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The simulation of ON-THE-FLY observations with the the bolometer arrays is a helpful tool to prepare your project, or to become familiar with the possibilities of the NIC software. We refer to the sections `Preparing Observations` and `NIC Language Internal Help` for the commands FIELD and SIMULATE. With option /NMB, command FIELD prepares an empty NMB file. Command SIMULATE then allows to write the ON-THE-FLY signals of a Gaussian model source (option /GAUSS), or for an equatorial image given by a GDF image (option /INPUT). Here, we describe to more detail an option that adds correlated noise to the simulated signals. Its syntax is

```
FIELD /NMB Scan_Number /NOISE Mean_rms Mean_corr
```

The resulting NMB file may be used to test algorithms for noise removal.

The algorithm

The generation of correlated fluctuations is a non-trivial task: The fluctuations of the noise in a bolometer channel are intercorrelated with those in the other ones, following some prescription for the correlation coefficient $r_{\rm ij}$ between two channels (i, j). We define the correlation coefficient by

$$r_{\rm ij} = \frac{n\sum_{\rm k} s_{\rm i,k}s_{\rm j,k} - \sum_{\rm k} s_{\rm i,k}\sum_{\rm k} s_{\rm j,k}}{\sqrt{\left(n\sum_{\rm k} s_{\rm i,k}^2 - (\sum_{\rm k} s_{\rm i,k})^2\right)\left(n\sum_{\rm k} s_{\rm j,k}^2 - (\sum_{\rm k} s_{\rm j,k})^2\right)}} \qquad (1)$$

where $s_{\rm i,k}$ is the kth signal record $(k = 1, ..., n)$ for channel i, and where n is the number of records considered (e.g. a whole scan, or a subscan). For a series of data of zero mean, $r_{\rm ij}$ is simply given by the scalar product of two vectors $\vec{s}_{\rm i}$ and $\vec{s}_{\rm j}$, each of unit length:

$$r_{\rm ij} = \vec{s}_{\rm i} \cdot \vec{s}_{\rm j}\,. \qquad (2)$$

It is possible to construct multivariate random variables $\vec{s}$ from a linear combination of univariate random variables $\vec{e}$. In $n-$dimensional space, we can generate $m \leq n$ such multivariate random variables, where m is the number of channels, i.e.

$$
\begin{aligned}
\vec{s}_{\mathrm{1}} &= a_{11}\vec{e}_1 \\
\vec{s}_{\mathrm{2}} &= a_{21}\vec{e}_1 + a_{22}\vec{e}_2 \\
\vec{s}_{\mathrm{3}} &= a_{31}\vec{e}_1 + a_{32}\vec{e}_2 + a_{33}\vec{e}_3 \\
&\vdots \\
\vec{s}_{\mathrm{m}} &= a_{\mathrm{m1}}\vec{e}_1 + a_{\mathrm{m2}}\vec{e}_2 + a_{\mathrm{m3}}\vec{e}_3 + \ldots + a_{\mathrm{mm}}\vec{e}_{\mathrm{m}} \qquad (3)
\end{aligned}
$$

There are $m(m+1)/2$ unknowns. $m(m-1)/2$ unknowns $a_{\mathrm{ij}}(i \neq j)$ may be determined from the correlation coefficients r_{ij}, the remaining m unknowns a_{ii} are given by the unity constraint, i.e. $\vec{s}_{\mathrm{i}} \cdot \vec{s}_{\mathrm{i}} = 1$. If two multivariate random variables of zero mean are uncorrelated, the two vectors representing them are almost perpendicular (exactly perpendicular only in ∞-dimensional space). The following listing is a pseudo-code formulation of the algorithm:

$a_{11} = 1$
For $j = 1$ to $m - 1$ {
    For $i = j + 1$ to m {
        $a_{\mathrm{ij}} = r_{\mathrm{ij}}$
        For $k = 1$ to $j - 1$: $a_{\mathrm{ij}} \leftarrow a_{\mathrm{ij}} - a_{\mathrm{ik}}a_{\mathrm{jk}}$
        $a_{\mathrm{ij}} \leftarrow a_{\mathrm{ij}}/a_{\mathrm{jj}}$
        $a_{\mathrm{j+1,j+1}} = 1$
        For $k = 1$ to j: $a_{\mathrm{j+1,j+1}} \leftarrow a_{\mathrm{j+1,j+1}} - a^2_{\mathrm{j+1,k}}$
        $a_{\mathrm{j+1,j+1}} \leftarrow \sqrt{a_{\mathrm{j+1,j+1}}}$
    }
}

Once the matrix $\{a_{\mathrm{ij}}\}$ is determined, you can construct a set of m multivariate random variables by applying equation 3. The resulting intercorrelated signals $\vec{s}_{\mathrm{i}}$ $(i = 1, ..., m)$ are vectors of n elements each and of unit length. The signals are then scaled to the desired standard deviation σ_{rms} (parameter *Mean_Rms* of option /NOISE) by

$$\vec{s}_{\mathrm{i}} \leftarrow \sigma_{\mathrm{rms}}\vec{s}_{\mathrm{i}}\sqrt{n}\,. \qquad (4)$$

In praxis, the strength of the correlation is determined by the coupling of the channels' near-field beam pattern to the fluctuations of the atmospheric emission. These fluctuations are due to turbulence in the troposphere, and are characterized by a structure function (Kolmogorov-law, the same law holds for fluctuations of the refractive index and of the emissivity). The observed noise correlation depends on the channel distance, on the mean correlation length of the turbulence, on the height of the emitting tropospheric layer, and on the wind speed. However, because the atmospheric emission is a weighted average across the channels' near fields, the Kolmogorov law cannot be directly deduced from the noise correlation law.

As a reasonable approach, the simulation in NIC uses

$$r_{\mathrm{ij}} = r_0 \cdot \exp\left(-\frac{d_{\mathrm{ij}}}{d_{\mathrm{min}}}\right) \tag{5}$$

where r_0 is the correlation coefficient (parameter *Mean_Corr* of option /NOISE) for the smallest channel distance d_{min} of the array, and where d_{ij} is the angular distance between two channels i, j.