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GILDAS
SOFTWARE

$$T(i)$$

$$f^{\text{obs}}(i) = f^{\text{obs}}_{\text{tuned}} + (i - i_0)\,\delta f^{\text{obs}},$$

f^{obs}

$$f^{\mathrm{obs}}_{\mathrm{tuned}}$$

i_0

δf^{obs}

$$\frac{v^{\text{obs}}}{c} = \frac{f^{\text{rest}} - f^{\text{obs}}}{f^{\text{rest}}} = 1 - \frac{f^{\text{obs}}}{f^{\text{rest}}},$$

v^{obs}

$$f^{\text{rest}}$$

$$d^{\mathrm{obs}}$$

$$\frac{f^{\mathrm{obs}}}{f^{\mathrm{rest}}} = 1 + d^{\mathrm{obs}} \quad \text{with} \quad d^{\mathrm{obs}} \equiv -\frac{v^{\mathrm{obs}}}{c}.$$

$$f^{\mathrm{rest}} = \frac{f^{\mathrm{obs}}}{1 + d^{\mathrm{obs}}}.$$

$v_{\rm sys}^{\rm obs}$

$$f^{\mathrm{rest}}(i) = f^{\mathrm{rest}}_{\mathrm{tuned}} + (i - i_0)\,\delta f^{\mathrm{rest}} \quad \text{with} \quad \delta f^{\mathrm{rest}} = \frac{\delta f^{\mathrm{obs}}}{1 + d^{\mathrm{obs}}_{\mathrm{sys}}} \quad \text{and} \quad d^{\mathrm{obs}}_{\mathrm{sys}} = -\frac{v^{\mathrm{obs}}_{\mathrm{sys}}}{c}.$$

$$f_{\text{tuned}}^{\text{rest}}$$

$$v_{\rm sys}^{\rm obs}$$

$$\frac{f_{\text{tuned}}^{\text{obs}}}{f_{\text{tuned}}^{\text{rest}}} = 1 + d_{\text{sys}}^{\text{obs}} \quad \text{with} \quad d_{\text{sys}}^{\text{obs}} \equiv -\frac{v_{\text{sys}}^{\text{obs}}}{c}.$$

$$f^{\mathrm{obs}}(i) - f^{\mathrm{obs}}_{\mathrm{tuned}}$$

$$\delta v^{\rm obs}(i) \equiv v^{\rm obs}(i) - v^{\rm obs}_{\rm sys}\,.$$

$$\frac{f^{\mathrm{obs}}(i)}{f^{\mathrm{rest}}_{\mathrm{tuned}}} = 1 + d^{\mathrm{obs}}(i) \quad \text{with} \quad d^{\mathrm{obs}}(i) \equiv -\frac{v^{\mathrm{obs}}(i)}{c}.$$

$$\delta v^{\mathrm{obs}}(i) = c\,\frac{f^{\mathrm{obs}}_{\mathrm{tuned}} - f^{\mathrm{obs}}(i)}{f^{\mathrm{rest}}_{\mathrm{tuned}}},$$

$$v^{\mathrm{obs}}(i) = v^{\mathrm{obs}}_{\mathrm{sys}} + (i - i_0)\,\delta v^{\mathrm{obs}} \quad \text{with} \quad \delta v^{\mathrm{obs}} = -c\frac{\delta f^{\mathrm{obs}}}{f^{\mathrm{rest}}_{\mathrm{tuned}}}.$$

$f_{\mathrm{LO}}^{\mathrm{obs}}$

$$f_{\mathrm{RF}}^{\mathrm{obs}}$$

$$f_{\mathrm{IF}}^{\mathrm{obs}}$$

$$f_{\mathrm{IF}}^{\mathrm{obs}}(i) = SB_{\mathrm{sign}}\ \left[f_{\mathrm{RF}}^{\mathrm{obs}}(i) - f_{\mathrm{LO}}^{\mathrm{obs}}\right]$$

$$SB_{\text{sign}} = -1$$

$$SB_{\text{sign}} = +1$$

$$f_{\mathrm{IF}}^{\mathrm{obs}}(i) = f_{\mathrm{IFtuned}}^{\mathrm{obs}} + (i - i_0)\,\delta f_{\mathrm{IF}}^{\mathrm{obs}},$$

$$f^{\mathrm{obs}}_{\mathrm{IFtuned}} \equiv SB_{\mathrm{sign}}\,(f^{\mathrm{obs}}_{\mathrm{RFtuned}} - f^{\mathrm{obs}}_{\mathrm{LO}}) \equiv \mathrm{cste.}$$

$f^{\mathrm{obs}}_{\mathrm{IFtuned}}$

$$f_{\mathrm{RF}}^{\mathrm{obs}}(i) = (f_{\mathrm{LO}}^{\mathrm{obs}} + SB_{\mathrm{sign}}\, f_{\mathrm{IFtuned}}^{\mathrm{obs}}) + SB_{\mathrm{sign}}\,(i - i_0)\,\delta f_{\mathrm{IF}}^{\mathrm{obs}}.$$

$f^{\text{obs}}_{\text{RFtuned}}$

$$f_{\mathrm{sig}}^{\mathrm{obs}}$$

$$f_{\mathrm{ima}}^{\mathrm{obs}}$$

$$f^{\mathrm{obs}}_{\mathrm{sig}}(i) = f^{\mathrm{obs}}_{\mathrm{sig,tuned}} + SB_{\mathrm{sign}}\,(i - i_0)\,\delta f^{\mathrm{obs}}_{\mathrm{IF}} \quad \text{with} \quad f^{\mathrm{obs}}_{\mathrm{sig,tuned}} \equiv f^{\mathrm{obs}}_{\mathrm{LO}} + SB_{\mathrm{sign}}\, f^{\mathrm{obs}}_{\mathrm{IFtuned}},$$

$$f^{\rm obs}_{\rm ima}(i) = f^{\rm obs}_{\rm ima,tuned} - SB_{\rm sign}\,(i - i_0)\,\delta f^{\rm obs}_{\rm IF} \quad \text{with} \quad f^{\rm obs}_{\rm ima,tuned} \equiv f^{\rm obs}_{\rm LO} - SB_{\rm sign}\, f^{\rm obs}_{\rm IFtuned}.$$

$$SB_{\rm sign}\,\delta f_{\rm IF}^{\rm obs}$$

SB_{sign}

$$\delta f_{\mathrm{IF}}^{\mathrm{obs}}$$

$$\delta f_{\mathrm{IF}}^{\mathrm{obs}} > 0$$

$$f_{\mathrm{sig}}^{\mathrm{obs}}(i) = f_{\mathrm{sig,tuned}}^{\mathrm{obs}} + (i - i_0)\,\delta f_{\mathrm{IF}}^{\mathrm{obs}} \quad \text{with} \quad f_{\mathrm{sig,tuned}}^{\mathrm{obs}} \equiv f_{\mathrm{LO}}^{\mathrm{obs}} + SB_{\mathrm{sign}}\, f_{\mathrm{IFtuned}}^{\mathrm{obs}},$$

$$f_{\rm ima}^{\rm obs}(i) = f_{\rm ima,tuned}^{\rm obs} - (i - i_0)\,\delta f_{\rm IF}^{\rm obs} \quad \text{with} \quad f_{\rm ima,tuned}^{\rm obs} \equiv f_{\rm LO}^{\rm obs} - SB_{\rm sign}\, f_{\rm IFtuned}^{\rm obs}.$$

$$\delta f_{\mathrm{IF}}^{\mathrm{obs}} > 0.$$

$$f_{\text{sig}}^{\text{rest}}(i) = f_{\text{sig,tuned}}^{\text{rest}} + (i - i_0)\, \delta f^{\text{rest}},$$

$$f_{\text{ima}}^{\text{rest}}(i) = f_{\text{ima,tuned}}^{\text{rest}} - (i - i_0)\,\delta f^{\text{rest}},$$

$$\delta f^{\mathrm{rest}} = \frac{\delta f_{\mathrm{IF}}^{\mathrm{obs}}}{1 + d_{\mathrm{sys}}^{\mathrm{obs}}} \quad \text{and} \quad d_{\mathrm{sys}}^{\mathrm{obs}} = -\frac{v_{\mathrm{sys}}^{\mathrm{obs}}}{c}.$$

$$f^{\mathrm{rest}}_{\mathrm{sig,tuned}}$$

$f_{\mathrm{ima,tuned}}^{\mathrm{rest}}$

$$SB^{\rm obs}_{\rm sep} \equiv SB_{\rm sign}\left[f^{\rm obs}_{\rm sig,tuned} - f^{\rm obs}_{\rm ima,tuned}\right].$$

$$SB_{\rm sep}^{\rm obs} = {\rm cste} = 2\, f_{\rm IFtuned}^{\rm obs}.$$

$$\frac{f^{\mathrm{obs}}_{\mathrm{sig,tuned}}}{f^{\mathrm{rest}}_{\mathrm{sig,tuned}}} = 1 + d^{\mathrm{obs}}_{\mathrm{sys}} \qquad \frac{f^{\mathrm{obs}}_{\mathrm{ima,tuned}}}{f^{\mathrm{rest}}_{\mathrm{ima,tuned}}} = 1 + d^{\mathrm{obs}}_{\mathrm{sys}}.$$

$$SB_{\rm sep}^{\rm rest} \equiv SB_{\rm sign}\left[f_{\rm sig,tuned}^{\rm rest} - f_{\rm ima,tuned}^{\rm rest}\right].$$

$$SB_{\rm sep}^{\rm obs} = SB_{\rm sep}^{\rm rest}\ \left[1 + d_{\rm sys}^{\rm obs}\right],$$

$$f^{\mathrm{rest}}_{\mathrm{ima,tuned}} = f^{\mathrm{rest}}_{\mathrm{sig,tuned}} - \frac{SB^{\mathrm{obs}}_{\mathrm{sep}}}{SB_{\mathrm{sign}}\left[1 + d^{\mathrm{obs}}_{\mathrm{sys}}\right]}.$$

$$f^{\mathrm{lsr}} = f^{\mathrm{obs}} + f^{\mathrm{rest}}\,\frac{v_{\mathrm{sys}}^{\mathrm{obs}} - v_{\mathrm{sys}}^{\mathrm{lsr}}}{c},$$

$$f^{\mathrm{obs}} = f^{\mathrm{rest}} \left(1 - \frac{v_{\mathrm{sys}}^{\mathrm{obs}}}{c}\right) \quad \text{and} \quad f^{\mathrm{lsr}} = f^{\mathrm{rest}} \left(1 - \frac{v_{\mathrm{sys}}^{\mathrm{lsr}}}{c}\right).$$

$$\left[SB_{\mathrm{sign}}\, f^{\mathrm{rest}}\, (v_{\mathrm{sys}}^{\mathrm{obs}} - v_{\mathrm{sys}}^{\mathrm{lsr}})/c\right]$$

$$f^{\mathrm{lsr}}_{\mathrm{sig,tuned}} = f^{\mathrm{obs}}_{\mathrm{sig,tuned}} + f^{\mathrm{rest}}_{\mathrm{sig,tuned}} \, \frac{v^{\mathrm{obs}}_{\mathrm{sys}} - v^{\mathrm{lsr}}_{\mathrm{sys}}}{c}.$$

$$f^{\mathrm{lsr}}(i) = f^{\mathrm{obs}}(i) + f^{\mathrm{rest}}_{\mathrm{sig,tuned}} \frac{v^{\mathrm{obs}}_{\mathrm{sys}} - v^{\mathrm{lsr}}_{\mathrm{sys}}}{c},$$

$$f^{\mathrm{lsr}}(i) = f^{\mathrm{obs}}(i) + f^{\mathrm{rest}}(i)\,\frac{v_{\mathrm{sys}}^{\mathrm{obs}} - v_{\mathrm{sys}}^{\mathrm{lsr}}}{c}.$$

$$\Delta f^{\rm lsr}(i) = (i - i_0)\,\delta f^{\rm rest}\,\frac{v_{\rm sys}^{\rm obs} - v_{\rm sys}^{\rm lsr}}{c}.$$

$$i = i_0$$

$f_{\text{tuned,sig}}^{\text{rest}}$

$$f^{\text{rest}}_{\text{tuned,ima}}$$

$$\delta f^{\mathrm{meas}}$$

$\mathrm{km\,s^{-1}}$

δv^{meas}

$v_{\mathrm{sys}}^{\mathrm{meas}}$

$$d_{\rm sys}^{\rm meas}$$

$$f_{\text{sig}}^{\text{rest}}(i) = f_{\text{sig,tuned}}^{\text{rest}} + (i - i_0)\,\delta f^{\text{rest}},$$

$$f_{\mathrm{ima}}^{\mathrm{rest}}(i) = f_{\mathrm{ima,tuned}}^{\mathrm{rest}} - (i - i_0)\,\delta f^{\mathrm{rest}},$$

$$\delta f^{\mathrm{rest}} = \frac{\delta f_{\mathrm{IF}}^{\mathrm{meas}}}{1 + d_{\mathrm{sys}}^{\mathrm{meas}}} \quad \text{and} \quad d_{\mathrm{sys}}^{\mathrm{meas}} = -\frac{v_{\mathrm{sys}}^{\mathrm{meas}}}{c},$$

$\delta f_{\mathrm{IF}}^{\mathrm{meas}}$

$v_{\text{sys}}^{\text{meas}}$

$$v^{\mathrm{meas}}(i) = v_{\mathrm{sys}}^{\mathrm{meas}} + (i - i_0)\,\delta v^{\mathrm{meas}} \quad \text{with} \quad \delta v^{\mathrm{meas}} = -c\frac{\delta f^{\mathrm{meas}}}{f_{\mathrm{tuned}}^{\mathrm{rest}}}.$$

$$v^{\mathrm{lsr}}(i) = v^{\mathrm{lsr}}_{\mathrm{sys}} + (i - i_0)\,\delta v^{\mathrm{lsr}} \quad \text{with} \quad \delta v^{\mathrm{lsr}} = -c\frac{\delta f^{\mathrm{lsr}}}{f^{\mathrm{rest}}_{\mathrm{tuned}}}.$$

$$v^{\mathrm{lsr}}(i) = v^{\mathrm{lsr}}_{\mathrm{sys}} + (i - i_0)\,\delta v^{\mathrm{obs}} \quad \text{with} \quad \delta v^{\mathrm{obs}} = -c\frac{\delta f^{\mathrm{obs}}}{f^{\mathrm{rest}}_{\mathrm{tuned}}}.$$

$$\frac{\delta v^{\mathrm{obs}} - \delta v^{\mathrm{lsr}}}{\delta v^{\mathrm{lsr}}} = \frac{\delta f^{\mathrm{obs}}}{\delta f^{\mathrm{lsr}}} - 1 = \frac{1 + d_{\mathrm{sys}}^{\mathrm{lsr}}}{1 + d_{\mathrm{sys}}^{\mathrm{obs}}} - 1.$$

v/c

$$\frac{\delta v^{\mathrm{obs}} - \delta v^{\mathrm{lsr}}}{\delta v^{\mathrm{lsr}}} = \frac{v_{\mathrm{sys}}^{\mathrm{obs}} - v_{\mathrm{sys}}^{\mathrm{lsr}}}{c} \sim 10^{-4}.$$

$\sim 250\,\mathrm{km\,s^{-1}}$

mm

kHz

$$\delta f^{\mathrm{lsr}} = \delta f^{\mathrm{obs}}\,\frac{1 + d^{\mathrm{obs}}_{\mathrm{sys}}}{1 + d^{\mathrm{lsr}}_{\mathrm{sys}}}, \quad \delta v^{\mathrm{lsr}} = -c\frac{\delta f^{\mathrm{lsr}}}{f^{\mathrm{rest}}_{\mathrm{tuned}}}, \quad \text{and} \quad d^{\mathrm{lsr}} = 1 - \frac{v^{\mathrm{lsr}}}{c}.$$

$$f_{\mathrm{sig}}^{\mathrm{rest}}(i)$$

=

$$f^{\mathrm{rest}}_{\mathrm{sig,old}} + (i - i_{0,\mathrm{old}})\,\delta f^{\mathrm{rest}},$$

$$f^{\mathrm{rest}}_{\mathrm{sig,new}} + (i - i_{0,\mathrm{new}})\,\delta f^{\mathrm{rest}},$$

$$\delta f^{\mathrm{rest}} = \frac{\delta f^{\mathrm{meas}}_{\mathrm{IF}}}{1 + d^{\mathrm{meas}}_{\mathrm{sys}}}.$$

$$i_{0,\mathrm{new}} = i_{0,\mathrm{old}} + \frac{f^{\mathrm{rest}}_{\mathrm{sig,new}} - f^{\mathrm{rest}}_{\mathrm{sig,old}}}{\delta f^{\mathrm{meas}}_{\mathrm{IF}}} \left(1 + d^{\mathrm{meas}}_{\mathrm{sys}}\right)$$

$f_{\text{ima}}^{\text{rest}}(i)$

$$f_{\mathrm{ima,old}}^{\mathrm{rest}} - (i - i_{0,\mathrm{old}})\,\delta f^{\mathrm{rest}},$$

$$f^{\text{rest}}_{\text{ima,new}} - (i - i_{0,\text{new}})\,\delta f^{\text{rest}},$$

$f_{\mathrm{sig,new}}^{\mathrm{rest}}$

$$f_{\mathrm{sig,old}}^{\mathrm{rest}} + \left(i_{0,\mathrm{new}} - i_{0,\mathrm{old}}\right) \delta f^{\mathrm{rest}},$$

$$f_{\mathrm{ima,new}}^{\mathrm{rest}}$$

$$f^{\text{rest}}_{\text{ima,old}} - (i_{0,\text{new}} - i_{0,\text{old}})\,\delta f^{\text{rest}}.$$

$$f_{\mathrm{ima,new}}^{\mathrm{rest}} + f_{\mathrm{sig,new}}^{\mathrm{rest}} = f_{\mathrm{sig,old}}^{\mathrm{rest}} + f_{\mathrm{ima,old}}^{\mathrm{rest}}.$$

$$\delta v_{\mathrm{new}}^{\mathrm{meas}} = -c\frac{\delta f^{\mathrm{meas}}}{f_{\mathrm{sig,new}}^{\mathrm{rest}}}.$$

$$f^{\mathrm{meas}} = (1 + d^{\mathrm{meas}}_{\mathrm{old}})\, f^{\mathrm{rest}}_{\mathrm{old}} = (1 + d^{\mathrm{meas}}_{\mathrm{new}})\, f^{\mathrm{rest}}_{\mathrm{new}},$$

$$d^{\mathrm{meas}}_{\mathrm{old}} = -\frac{v^{\mathrm{meas}}_{\mathrm{sys,old}}}{c} \quad \text{and} \quad d^{\mathrm{meas}}_{\mathrm{new}} = -\frac{v^{\mathrm{meas}}_{\mathrm{sys,new}}}{c}.$$

$$f_{\text{new}}^{\text{rest}} = f_{\text{old}}^{\text{rest}} \frac{1 + d_{\text{old}}^{\text{meas}}}{1 + d_{\text{new}}^{\text{meas}}}.$$

$$f_{\text{sig,new}}^{\text{rest}}(i)$$

$$f^{\mathrm{rest}}_{\mathrm{sig,tuned}} + (i - i_{0,\mathrm{new}})\,\delta f^{\mathrm{rest}}_{\mathrm{new}}$$

$$\left[f_{\mathrm{sig,tuned}}^{\mathrm{rest}} + (i - i_{0,\mathrm{old}})\, \delta f_{\mathrm{old}}^{\mathrm{rest}}\right] \frac{1 + d_{\mathrm{old}}^{\mathrm{meas}}}{1 + d_{\mathrm{new}}^{\mathrm{meas}}}.$$

$$i_{0,\mathrm{new}} = i_{0,\mathrm{old}} + \frac{f^{\mathrm{rest}}_{\mathrm{sig,tuned}}}{\delta f^{\mathrm{meas}}_{\mathrm{old}}} \left[d^{\mathrm{meas}}_{\mathrm{new}} - d^{\mathrm{meas}}_{\mathrm{old}}\right],$$

$$\delta f_{\text{new}}^{\text{rest}} = \delta f_{\text{old}}^{\text{rest}} \frac{1 + d_{\text{old}}^{\text{meas}}}{1 + d_{\text{new}}^{\text{meas}}}, \quad i.e., \quad \delta f_{\text{new}}^{\text{meas}} = \delta f_{\text{old}}^{\text{meas}}.$$

$$f_{\text{ima,new}}^{\text{rest}}(i)$$

$$f^{\text{rest}}_{\text{ima,tuned,new}} - (i - i_{0,\text{new}})\,\delta f^{\text{rest}}_{\text{new}}$$

$$\left[f_{\mathrm{ima,tuned,old}}^{\mathrm{rest}} - (i - i_{0,\mathrm{old}})\,\delta f_{\mathrm{old}}^{\mathrm{rest}}\right]\;\frac{1 + d_{\mathrm{old}}^{\mathrm{meas}}}{1 + d_{\mathrm{new}}^{\mathrm{meas}}}.$$

$$f_{\mathrm{ima,new}}^{\mathrm{rest}} = f_{\mathrm{ima,old}}^{\mathrm{rest}} \frac{1 + d_{\mathrm{old}}^{\mathrm{meas}}}{1 + d_{\mathrm{new}}^{\mathrm{meas}}} - f_{\mathrm{sig,tuned}}^{\mathrm{rest}} \frac{d_{\mathrm{new}}^{\mathrm{meas}} - d_{\mathrm{old}}^{\mathrm{meas}}}{1 + d_{\mathrm{new}}^{\mathrm{meas}}}.$$

$$d_{\rm old}^{\rm meas} = -\frac{v_{\rm lsr}^{\rm meas} + v_{\rm sys,old}^{\rm lsr}}{c} \quad \text{and} \quad d_{\rm new}^{\rm meas} = -\frac{v_{\rm lsr}^{\rm meas} + v_{\rm sys,new}^{\rm lsr}}{c}.$$

$$d^{\rm meas}_{\rm new} = d^{\rm meas}_{\rm old} + \frac{v^{\rm lsr}_{\rm sys,old} - v^{\rm lsr}_{\rm sys,new}}{c}.$$

$$f^{\text{obs}} = f^{\text{rest}}\,\frac{\sqrt{c^2 - {v^{\text{obs}}}^2}}{c + v_{\parallel}^{\text{obs}}},$$

$$v^{\mathrm{obs}\,2} = v_{\parallel}^{\mathrm{obs}\,2} + v_{\perp}^{\mathrm{obs}\,2}$$

$v_{\parallel}^{\mathrm{obs}}$

$$v_{\perp}^{\mathrm{obs}}$$

$$v_{\parallel}^{\mathrm{obs}} \gg v_{\perp}^{\mathrm{obs}}$$

$$f^{\mathrm{obs}} = f^{\mathrm{rest}} \sqrt{\frac{c - v_{\parallel}^{\mathrm{obs}}}{c + v_{\parallel}^{\mathrm{obs}}}}.$$

$$f^{\mathrm{obs}} = f^{\mathrm{rest}} \left(1 - \frac{v_{\parallel}^{\mathrm{obs}}}{c}\right).$$

$$v^{\mathrm{obs}}(i) = v^{\mathrm{obs}}_{\mathrm{sys}} + (i - i_0)\,\delta v^{\mathrm{obs}} \quad \text{with} \quad \delta v^{\mathrm{obs}} = -c\frac{\delta f^{\mathrm{obs}}}{f^{\mathrm{rest}}_{\mathrm{tuned}}}.$$

$$\lambda^{\mathrm{obs}} = \lambda^{\mathrm{rest}} \sqrt{\frac{c + v_{\parallel}^{\mathrm{obs}}}{c - v_{\parallel}^{\mathrm{obs}}}}.$$

$$\lambda^{\mathrm{obs}} = \lambda^{\mathrm{rest}} \left(1 + \frac{v_{\parallel}^{\mathrm{obs}}}{c}\right).$$

$$f^{\mathrm{obs}} = \frac{f^{\mathrm{rest}}}{1 + \frac{v_{\parallel}^{\mathrm{obs}}}{c}}.$$

$$v_{\rm opt}^{\rm obs}(i) = v_{\rm sys,opt}^{\rm obs} + (i - i_0)\,\delta v_{\rm opt}^{\rm obs} \quad \text{with} \quad \delta v_{\rm opt}^{\rm obs} = +c\,\frac{\delta\lambda^{\rm obs}}{\lambda_{\rm tuned}^{\rm rest}}.$$

v_1

$$f^{\mathrm{obs}}_{\mathrm{tuned,conv1}} = f^{\mathrm{rest}}_{\mathrm{tuned}}\, D_1(v_1),$$

$D_1()$

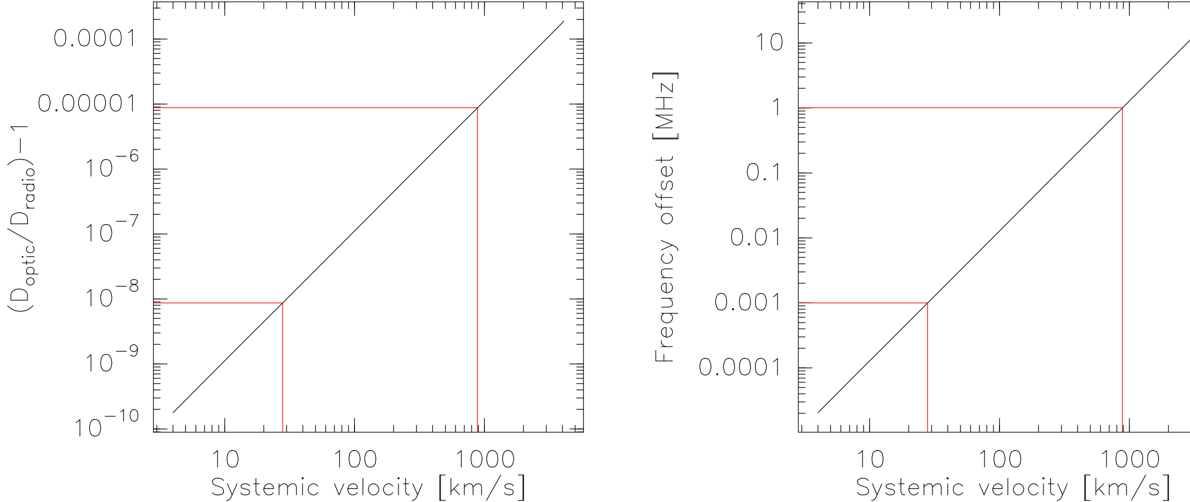
$$D_{\mathrm{opt}}(v_{\mathrm{opt}}) = \frac{1}{1 + \frac{v_{\mathrm{opt}}}{c}}.$$

$$f_{\text{tuned},1}^{\text{obs}}$$

$D_2()$

$$f^{\text{rest}}(i) = f^{\text{rest}}_{\text{apparent}} + (i - i_0)\,\delta f^{\text{rest}}_{\text{apparent}},$$

$$f^{\mathrm{rest}}_{\mathrm{apparent}} = f^{\mathrm{rest}}_{\mathrm{tuned}}\,\frac{D_1(v_1)}{D_2(v_1)}, \quad \text{and} \quad \delta f^{\mathrm{rest}}_{\mathrm{apparent}} = \delta f^{\mathrm{rest}}_{\mathrm{tuned}}\,\frac{D_1(v_1)}{D_2(v_1)}.$$



0.0001
0.00001
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-10}
$(D_{optic}/D_{radio})-1$
10
100
1000
Systemic velocity [km/s]
10
1
0.1
0.01
0.001
0.0001
Frequency offset [MHz]
10
100
1000
Systemic velocity [km/s]

10^{-5}

$$f^{\mathrm{rest}}_{\mathrm{sig,new}} = f^{\mathrm{rest}}_{\mathrm{sig,old}} \, \frac{D_{\mathrm{optic}}(v^{\mathrm{obs}}_{\mathrm{sys,old}})}{D_{\mathrm{radio}}(v^{\mathrm{obs}}_{\mathrm{sys,new}})}, \quad f^{\mathrm{rest}}_{\mathrm{ima,new}} = f^{\mathrm{rest}}_{\mathrm{ima,old}} \, \frac{D_{\mathrm{optic}}(v^{\mathrm{obs}}_{\mathrm{sys,old}})}{D_{\mathrm{radio}}(v^{\mathrm{obs}}_{\mathrm{sys,new}})},$$

$$\delta f_{\mathrm{new}}^{\mathrm{meas}} = \delta f_{\mathrm{old}}^{\mathrm{meas}} \, \frac{D_{\mathrm{optic}}(v_{\mathrm{sys,old}}^{\mathrm{obs}})}{D_{\mathrm{radio}}(v_{\mathrm{sys,new}}^{\mathrm{obs}})}, \quad \text{and} \quad \delta v_{\mathrm{new}}^{\mathrm{meas}} = -c \frac{\delta f_{\mathrm{new}}^{\mathrm{meas}}}{f_{\mathrm{sig,new}}^{\mathrm{rest}}}.$$

$f_{\mathrm{sig,new}}^{\mathrm{rest}}$

$$f_{\text{ima,new}}^{\text{rest}}$$

$$\delta f_{\rm new}^{\rm meas}$$

$$f^{\mathrm{rest}}_{\mathrm{sig,new}} = f^{\mathrm{rest}}_{\mathrm{sig,old}} \, \frac{D_{\mathrm{relat}}(v^{\mathrm{obs}}_{\mathrm{sys,old}})}{D_{\mathrm{relat}}(v^{\mathrm{obs}}_{\mathrm{sys,new}})}, \quad f^{\mathrm{rest}}_{\mathrm{ima,new}} = f^{\mathrm{rest}}_{\mathrm{ima,old}} \, \frac{D_{\mathrm{relat}}(v^{\mathrm{obs}}_{\mathrm{sys,old}})}{D_{\mathrm{relat}}(v^{\mathrm{obs}}_{\mathrm{sys,new}})},$$

$$\delta f_{\mathrm{new}}^{\mathrm{meas}} = \delta f_{\mathrm{old}}^{\mathrm{meas}} \, \frac{D_{\mathrm{relat}}(v_{\mathrm{sys,old}}^{\mathrm{obs}})}{D_{\mathrm{relat}}(v_{\mathrm{sys,new}}^{\mathrm{obs}})}, \quad \text{and} \quad \delta v_{\mathrm{new}}^{\mathrm{meas}} = -c \frac{\delta f_{\mathrm{new}}^{\mathrm{meas}}}{f_{\mathrm{sig,new}}^{\mathrm{rest}}}.$$

$$\frac{D_{\mathrm{relat}}(v_{\mathrm{sys,old}}^{\mathrm{obs}})}{D_{\mathrm{relat}}(v_{\mathrm{sys,new}}^{\mathrm{obs}})} = \frac{D_{\mathrm{relat}}(v_{\mathrm{sys,old}}^{\mathrm{lsr}})}{D_{\mathrm{relat}}(v_{\mathrm{sys,new}}^{\mathrm{lsr}})}.$$

$$f^{\rm rest}_{\rm sig,new} = f^{\rm rest}_{\rm sig,old}\,\frac{D_{\rm relat}(v^{\rm lsr}_{\rm sys,old})}{D_{\rm relat}(v^{\rm lsr}_{\rm sys,new})}, \quad f^{\rm rest}_{\rm ima,new} = f^{\rm rest}_{\rm ima,old}\,\frac{D_{\rm relat}(v^{\rm lsr}_{\rm sys,old})}{D_{\rm relat}(v^{\rm lsr}_{\rm sys,new})},$$

$$\delta f_{\rm new}^{\rm meas} = \delta f_{\rm old}^{\rm meas}\,\frac{D_{\rm relat}(v_{\rm sys,old}^{\rm lsr})}{D_{\rm relat}(v_{\rm sys,new}^{\rm lsr})}, \quad \text{and} \quad \delta v_{\rm new}^{\rm meas} = -c\frac{\delta f_{\rm new}^{\rm meas}}{f_{\rm sig,new}^{\rm rest}}.$$

$v_{\mathrm{sys}}^{\mathrm{lsr}}$