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Description of the spectral axis handling in CLASS

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Abstract

An observer is interested by the physical properties (line frequencies, velocity of the gas) of a source in its own reference frame. However, the observations happen in a different frame, namely the topocentric frame for ground-based observations. This implies a change of frame to infer the physical properties in the source frame from the measured frequencies in the measurement frame. This memo tries to explain the different interpretation of the spectral axis, the additional subtleties related to heterodyne receivers in radio-astronomy, the current description of the spectral axis handling in CLASS, and the impact of various short-comings that happen for historical reasons.

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1 Introduction

1.1 Spectrometers in radio-astronomy

Typical radio-astronomy spectrometers (*e.g.*, correlators or Fourier transform spectrometers) deliver the brightness along a given line of sight at regularly spaced frequencies *in the observatory frame*. The spectrum can be represented as a set of brightness temperatures $T(i)$, where the frequency axis in the observatory frame is defined as

$$f^{\rm obs}(i) = f^{\rm obs}_{\rm tuned} + (i - i_0)\,\delta f^{\rm obs}, \tag{1}$$

where $f^{\rm obs}(i)$ is the frequency at the channel i, $f^{\rm obs}_{\rm tuned}$ the frequency at the reference channel i_0, and $\delta f^{\rm obs}$ the channel spacing.

1.2 The Doppler effect

The Doppler effect is used to interpret the frequency axis of the measured spectrum. Using the radio convention, the non-relativistic Doppler effect in the observatory frame can be written as

$$\frac{v^{\rm obs}}{c} = \frac{f^{\rm rest} - f^{\rm obs}}{f^{\rm rest}} = 1 - \frac{f^{\rm obs}}{f^{\rm rest}}, \tag{2}$$

where $v^{\rm obs}$ is the velocity of the source in the observatory frame, $f^{\rm obs}$ and $f^{\rm rest}$ are the frequency of the measured photon in the observatory and rest frame respectively. $v^{\rm obs}$ is positive if the source recesses and the rest frame is defined as the frame where the velocity of the emitting gas cell is zero. Introducing the doppler parameter $d^{\rm obs}$, we obtain

$$\frac{f^{\rm obs}}{f^{\rm rest}} = 1 + d^{\rm obs} \quad \text{with} \quad d^{\rm obs} \equiv -\frac{v^{\rm obs}}{c}. \tag{3}$$

1.3 Interpretation 1: At fixed $v^{\rm obs}$

Let's assume that the same gas cell in the source emits two lines at different frequencies. We are at fixed $v^{\rm obs}$ and thus $d^{\rm obs}$ because we consider the same gas cell. The frequency axes in the rest and observatory frames are thus given by

$$f^{\rm rest} = \frac{f^{\rm obs}}{1 + d^{\rm obs}}. \tag{4}$$

This means that the frequency separation between the two lines is different in the rest and observatory frames. The modeller has easily access to the rest frame frequencies of the line. It is thus important to display the spectrum frequency axis in the rest frequency axis. This can be achieved only for one velocity (The reasoning is here done at fixed $v^{\rm obs}$), which is by default assumed to be the systemic velocity of the source in the observatory frame, *i.e.*, the mean velocity of the source gas in the observatory frame, written $v^{\rm obs}_{\rm sys}$.

As a convention, CLASS assumes that the tuned rest frequency and its corresponding observatory frequency at the source systemic velocity are associated to the common reference channel i_0. CLASS then displays the frequency axis in the rest frequency associated to the source systemic velocity through

$$f^{\rm rest}(i) = f^{\rm rest}_{\rm tuned} + (i - i_0)\,\delta f^{\rm rest} \quad \text{with} \quad \delta f^{\rm rest} = \frac{\delta f^{\rm obs}}{1 + d^{\rm obs}_{\rm sys}} \quad \text{and} \quad d^{\rm obs}_{\rm sys} = -\frac{v^{\rm obs}_{\rm sys}}{c}. \tag{5}$$

The plotted spectrum thus correctly displays the line at rest frequency positions, *i.e.*, the brightnesses of the gas whose velocity is equal to the systemic velocity in the observatory frame.

1.4 Interpretation 2: At fixed $f^{\rm rest}$

Let's assume that the measured spectrum is made of a line centered around $f^{\rm obs}_{\rm tuned}$ in the observatory frame. The associated rest frame frequency, *i.e.*, the frequency of the line at rest, will be $f^{\rm rest}_{\rm tuned}$. The observation is naturally set up so that the source frame is also the rest frame. This implies that $f^{\rm obs}_{\rm tuned}$, $f^{\rm rest}_{\rm tuned}$ and $v^{\rm obs}_{\rm sys}$ are linked through

$$\frac{f^{\rm obs}_{\rm tuned}}{f^{\rm rest}_{\rm tuned}} = 1 + d^{\rm obs}_{\rm sys} \quad \text{with} \quad d^{\rm obs}_{\rm sys} \equiv -\frac{v^{\rm obs}_{\rm sys}}{c}. \tag{6}$$

The modeller will use the Doppler effect to interpret the observed frequency difference $f^{\rm obs}(i) - f^{\rm obs}_{\rm tuned}$ as a local velocity difference (projected along the line of sight direction) around the systemic velocity of the source. Let's write $v^{\rm obs}(i)$ the velocity associated to $f^{\rm obs}(i)$. The velocity difference will be noted

$$\delta v^{\rm obs}(i) \equiv v^{\rm obs}(i) - v^{\rm obs}_{\rm sys}. \tag{7}$$

In order to derive this velocity difference, we associate the velocity to each observed frequency through the Doppler effect, *i.e.*,

$$\frac{f^{\rm obs}(i)}{f^{\rm rest}_{\rm tuned}} = 1 + d^{\rm obs}(i) \quad \text{with} \quad d^{\rm obs}(i) \equiv -\frac{v^{\rm obs}(i)}{c}. \tag{8}$$

It is easy to deduce that

$$\delta v^{\rm obs}(i) = c\,\frac{f^{\rm obs}_{\rm tuned} - f^{\rm obs}(i)}{f^{\rm rest}_{\rm tuned}}, \tag{9}$$

i.e., the local velocity difference is proportional to the observed frequency difference. We can define a linear velocity axis associated to the set of brightnesses, $T(i)$ as

$$v^{\rm obs}(i) = v^{\rm obs}_{\rm sys} + (i - i_0)\,\delta v^{\rm obs} \quad \text{with} \quad \delta v^{\rm obs} = -c\frac{\delta f^{\rm obs}}{f^{\rm rest}_{\rm tuned}}. \tag{10}$$

This velocity scale is only meaningful locally, i.e.*, as long as the brightnesses can be associated to the line rest frequency under consideration.*

2 Heterodyne receivers and signal/image frequency axes

2.1 USB and LSB frequency axes in the observatory frame

Heterodyne receivers mix a local oscillator frequency, $f^{\rm obs}_{\rm LO}$, to the astronomical signal to transfer the frequency axis from high radio frequencies, $f^{\rm obs}_{\rm RF}$, to low intermediate frequencies, $f^{\rm obs}_{\rm IF}$, which are then suitable for easy electronic processing. A consequence is that two bands of radio frequencies, namely the lower side band (LSB) and the upper side band (USB) are mixed in the observed spectrum, *i.e.*, the set of brightnesses, $T(i)$, are associated to one IF frequency axis and two RF frequency axes linked through

$$f^{\rm obs}_{\rm IF}(i) = SB_{\rm sign}\,\left[f^{\rm obs}_{\rm RF}(i) - f^{\rm obs}_{\rm LO}\right] \tag{11}$$

with $SB_{\rm sign} = -1$ for the RF frequency axis belonging to the lower side band (LSB) tuning and $SB_{\rm sign} = +1$ for the RF frequency axis belonging to the upper side band (USB). The measured spectra is thus in the IF axis and Eq. 1 must be modified as

$$f^{\rm obs}_{\rm IF}(i) = f^{\rm obs}_{\rm IFtuned} + (i - i_0)\,\delta f^{\rm obs}_{\rm IF}, \tag{12}$$

with

$$f^{\rm obs}_{\rm IFtuned} \equiv SB_{\rm sign}\,(f^{\rm obs}_{\rm RFtuned} - f^{\rm obs}_{\rm LO}) \equiv \text{cste}. \tag{13}$$

The constancy of $f^{\rm obs}_{\rm IFtuned}$ is a property of the heterodyne receivers. Combining the last 3 equations, we yield

$$f^{\rm obs}_{\rm RF}(i) = (f^{\rm obs}_{\rm LO} + SB_{\rm sign}\,f^{\rm obs}_{\rm IFtuned}) + SB_{\rm sign}\,(i - i_0)\,\delta f^{\rm obs}_{\rm IF}. \tag{14}$$

2.2 Signal and image frequency axes in the observatory frame

The RF tuning frequency, $f^{\rm obs}_{\rm RFtuned}$, can be choosen in either band. This band is called the signal band. The other band is then called the image band. The associated RF frequencies are written $f^{\rm obs}_{\rm sig}$ and $f^{\rm obs}_{\rm ima}$. We obtain

$$f^{\rm obs}_{\rm sig}(i) = f^{\rm obs}_{\rm sig,tuned} + SB_{\rm sign}\,(i-i_0)\,\delta f^{\rm obs}_{\rm IF} \quad \text{with} \quad f^{\rm obs}_{\rm sig,tuned} \equiv f^{\rm obs}_{\rm LO} + SB_{\rm sign}\,f^{\rm obs}_{\rm IFtuned}, \tag{15}$$

and

$$f^{\rm obs}_{\rm ima}(i) = f^{\rm obs}_{\rm ima,tuned} - SB_{\rm sign}\,(i-i_0)\,\delta f^{\rm obs}_{\rm IF} \quad \text{with} \quad f^{\rm obs}_{\rm ima,tuned} \equiv f^{\rm obs}_{\rm LO} - SB_{\rm sign}\,f^{\rm obs}_{\rm IFtuned}. \tag{16}$$

The sign of the product $SB_{\rm sign}\,\delta f^{\rm obs}_{\rm IF}$ is arbitrary. Indeed, it only defines how the brightnesses are ordered in memory, *i.e.*, whether the frequency increases or decreases when i increases. It thus is simpler to assume that this product is always positive. This implies that the signal frequency increases and the image frequency decreases when i increases. In other words, we adopt the convention that the signal brightnesses are ordered by increasing frequency when i increases, *i.e.*, the product $SB_{\rm sign}\,\delta f^{\rm obs}_{\rm IF}$ is always positive.

As the absolute value of $SB_{\rm sign}$ is 1, we will greatly simplify the equations by replacing $SB_{\rm sign}\,\delta f^{\rm obs}_{\rm IF}$ with $\delta f^{\rm obs}_{\rm IF}$ with the convention that $\delta f^{\rm obs}_{\rm IF} > 0$ in Eqs 15 and 16. We can then rewrite them as

$$f^{\rm obs}_{\rm sig}(i) = f^{\rm obs}_{\rm sig,tuned} + (i-i_0)\,\delta f^{\rm obs}_{\rm IF} \quad \text{with} \quad f^{\rm obs}_{\rm sig,tuned} \equiv f^{\rm obs}_{\rm LO} + SB_{\rm sign}\,f^{\rm obs}_{\rm IFtuned}, \tag{17}$$

and

$$f^{\rm obs}_{\rm ima}(i) = f^{\rm obs}_{\rm ima,tuned} - (i-i_0)\,\delta f^{\rm obs}_{\rm IF} \quad \text{with} \quad f^{\rm obs}_{\rm ima,tuned} \equiv f^{\rm obs}_{\rm LO} - SB_{\rm sign}\,f^{\rm obs}_{\rm IFtuned}. \tag{18}$$

with

$$\delta f^{\rm obs}_{\rm IF} > 0. \tag{19}$$

2.3 Signal and image frequency axes in the rest frame

The discussion of Section 1.3 is still valid for both the signal and image frequency axes. Eq. 5 can easily be rewritten

$$f^{\rm rest}_{\rm sig}(i) = f^{\rm rest}_{\rm sig,tuned} + (i-i_0)\,\delta f^{\rm rest}, \tag{20}$$

and

$$f^{\rm rest}_{\rm ima}(i) = f^{\rm rest}_{\rm ima,tuned} - (i-i_0)\,\delta f^{\rm rest}, \tag{21}$$

with

$$\delta f^{\rm rest} = \frac{\delta f^{\rm obs}_{\rm IF}}{1+d^{\rm obs}_{\rm sys}} \quad \text{and} \quad d^{\rm obs}_{\rm sys} = -\frac{v^{\rm obs}_{\rm sys}}{c}. \tag{22}$$

$f^{\rm rest}_{\rm sig,tuned}$ is simply the tuning frequency entered by the user. However, $f^{\rm rest}_{\rm ima,tuned}$ must be computed. The easiest way is to define the side band separation as

$$SB^{\rm obs}_{\rm sep} \equiv SB_{\rm sign}\left[f^{\rm obs}_{\rm sig,tuned} - f^{\rm obs}_{\rm ima,tuned}\right]. \tag{23}$$

It is straightfoward to show that it is constant and equal to

$$SB^{\rm obs}_{\rm sep} = \text{cste} = 2\,f^{\rm obs}_{\rm IFtuned}. \tag{24}$$

In the rest (source) frame, the following relations hold

$$\frac{f^{\rm obs}_{\rm sig,tuned}}{f^{\rm rest}_{\rm sig,tuned}} = 1 + d^{\rm obs}_{\rm sys} \qquad \frac{f^{\rm obs}_{\rm ima,tuned}}{f^{\rm rest}_{\rm ima,tuned}} = 1 + d^{\rm obs}_{\rm sys}. \tag{25}$$

The side band separation in the rest frame is defined as

$$SB^{\rm rest}_{\rm sep} \equiv SB_{\rm sign}\left[f^{\rm rest}_{\rm sig,tuned} - f^{\rm rest}_{\rm ima,tuned}\right]. \tag{26}$$

It is then straightforward to show that

$$SB_{\rm sep}^{\rm obs} = SB_{\rm sep}^{\rm rest}\,\left[1 + d_{\rm sys}^{\rm obs}\right], \tag{27}$$

and

$$f_{\rm ima,tuned}^{\rm rest} = f_{\rm sig,tuned}^{\rm rest} - \frac{SB_{\rm sep}^{\rm obs}}{SB_{\rm sign}\,\left[1 + d_{\rm sys}^{\rm obs}\right]}. \tag{28}$$

3 Earth movements

3.1 Periodic shift and dilatation of the rest frequency axis

For a Earth-based observatory, the observed velocity of the source changes with time due to the rotation of the Earth around its axis and around the sun. This implies that the change from the observatory to the rest frame varies with time. The frequency scale in the observatory frame is fixed by the hardware. The frequency scale in the rest frame thus changes with time in two different ways, a shift and a dilatation. The tuned rest frequency is shifted, while the frequency resolution dilates the frequency axis around the tuned frequency.

If nothing is done, averaging spectra to increase the signal-to-noise ratio would imply first a resampling to align the observed frequency scales. It is desirable to avoid this because resampling has many unwanted side-effects, as round-off errors or introduced correlations of the noise levels between channels (see IRAM Memo 2009-4). We present here hardware solutions to this problem.

3.2 Stopping the periodic shift

As the systemic velocity of the source in the observatory frame varies with time in a predictable way, the radio-observatories tune the local oscillator frequency in order to stop the shifting of the observed frequency scale. For instance, the relation between the observatory frequency and the LSR (Local Standard of Rest) frequency is

$$f^{\rm lsr} = f^{\rm obs} + f^{\rm rest}\,\frac{v_{\rm sys}^{\rm obs} - v_{\rm sys}^{\rm lsr}}{c}, \tag{29}$$

because

$$f^{\rm obs} = f^{\rm rest}\left(1 - \frac{v_{\rm sys}^{\rm obs}}{c}\right) \quad \text{and} \quad f^{\rm lsr} = f^{\rm rest}\left(1 - \frac{v_{\rm sys}^{\rm lsr}}{c}\right). \tag{30}$$

Thus adding $\left[SB_{\rm sign}\,f^{\rm rest}\,(v_{\rm sys}^{\rm obs} - v_{\rm sys}^{\rm lsr})/c\right]$ to the local oscillator frequency makes the signal frequency appear as if it was measured in the LSR frame. All the velocities are then expressed in the LSR frame instead of the observatory frame.

However, there is a single local oscillator frequency. Let's assume that the correction is done at the tuned signal frequency, *i.e.*,

$$f_{\rm sig,tuned}^{\rm lsr} = f_{\rm sig,tuned}^{\rm obs} + f_{\rm sig,tuned}^{\rm rest}\,\frac{v_{\rm sys}^{\rm obs} - v_{\rm sys}^{\rm lsr}}{c}. \tag{31}$$

The correction which is applied to another frequency is then

$$f^{\rm lsr}(i) = f^{\rm obs}(i) + f_{\rm sig,tuned}^{\rm rest}\,\frac{v_{\rm sys}^{\rm obs} - v_{\rm sys}^{\rm lsr}}{c}, \tag{32}$$

while the correction that should have been applied to another frequency is

$$f^{\rm lsr}(i) = f^{\rm obs}(i) + f^{\rm rest}(i)\,\frac{v_{\rm sys}^{\rm obs} - v_{\rm sys}^{\rm lsr}}{c}. \tag{33}$$

Table 1: CLASS header parameters used to describe the frequency/velocity axes and their translation in this document.

CLASS name	Unit	Translation	Notation
nchan	—	Number of channel	—
rchan	—	Reference channel	i_0
restf	MHz	Tuned signal rest frequency	$f^{\rm rest}_{\rm tuned,sig}$
image	MHz	Tuned image rest frequency	$f^{\rm rest}_{\rm tuned,ima}$
fres	MHz	Observatory channel spacing	$\delta f^{\rm meas}$
vres	$\rm km\,s^{-1}$	Velocity channel spacing	$\delta v^{\rm meas}$
voff	$\rm km\,s^{-1}$	source systemic velocity	$v^{\rm meas}_{\rm sys}$
doppler	—	Doppler factor	$d^{\rm meas}_{\rm sys}$

The difference (or error) is

$$\Delta f^{\rm lsr}(i) = (i - i_0)\,\delta f^{\rm rest}\,\frac{v^{\rm obs}_{\rm sys} - v^{\rm lsr}_{\rm sys}}{c}. \tag{34}$$

The correction is thus exact only in $i = i_0$, *i.e.*, for the tuned signal frequency[1]. All the other signal and image frequencies oscillates with time with a frequency amplitude which linearly increases with the distance to the reference channel. In other words, this corrects only the global frequency shift, not the dilatation around the reference channel.

3.3 Stopping the periodic dilatation

At IRAM-30m, G. Paubert proposes to stop the periodic dilatation in the rest frequency by adapting the channel spacing of the spectrometer frequency comb in real time. This in combination with LO tuning will stop all temporal variations of the frequency axis, removing the need for resampling when averaging spectra observed at different time of year.

This proposition is equal to a change of reference frame for the channel spacing. Due to hardware limitation, this change of frame can only be done from the observatory to the barycentric frame. This implies that adaptation of the frequency channel spacing when going from the barycentric frame to the source rest frame will still be needed.

4 CLASS implementation

4.1 The header parameters

Table 1 describes the CLASS header parameters used to describe the frequency/velocity axes and their translation in this document. The signal and image frequency axis are given in the source (rest) frame by

$$f^{\rm rest}_{\rm sig}(i) = f^{\rm rest}_{\rm sig,tuned} + (i - i_0)\,\delta f^{\rm rest}, \tag{35}$$

and

$$f^{\rm rest}_{\rm ima}(i) = f^{\rm rest}_{\rm ima,tuned} - (i - i_0)\,\delta f^{\rm rest}, \tag{36}$$

with

$$\delta f^{\rm rest} = \frac{\delta f^{\rm meas}_{\rm IF}}{1 + d^{\rm meas}_{\rm sys}} \quad \text{and} \quad d^{\rm meas}_{\rm sys} = -\frac{v^{\rm meas}_{\rm sys}}{c}, \tag{37}$$

[1] In fact, there often are several local oscillators in a radio-observatory. At the Plateau de Bure Interferometer, two of them are used to stop both the signal and image tuned frequencies. This was useful at the time when the 1mm receivers were double-sideband and the sideband separation was obtained with Walsh phase modulation/demodulation.

where $\delta f_{\rm IF}^{\rm meas}$ is the frequency channel spacing , and $v_{\rm sys}^{\rm meas}$ is the source systemic velocity *in the measurement frame*. The corresponding velocity axis in the measurement frame is thus given by

$$v^{\rm meas}(i) = v_{\rm sys}^{\rm meas} + (i - i_0)\,\delta v^{\rm meas} \quad \text{with} \quad \delta v^{\rm meas} = -c\frac{\delta f^{\rm meas}}{f_{\rm tuned}^{\rm rest}}. \tag{38}$$

4.2 The IRAM-30m case as of 2019-07-06: Systemic velocity in (slightly) different frames

The previous equations assume that the systemic velocity is given in the same frame as the frequency channel spacing. However, at the 30m the systemic velocity is usually given in the LSR frame (as requested by the user) and the channel spacing and the doppler factor are given in the observatory frame.

On one hand, the rest frequency scale is correctly computed by `CLASS` because the channel spacing and the doppler factor are given in the same frame. On the other hand, the velocity scale is only approximated because the source systemic velocity is given in another frame, most often the LSR frame.

The end-user (astronomer) is interested with the velocity scale in the LSR frame

$$v^{\rm lsr}(i) = v_{\rm sys}^{\rm lsr} + (i - i_0)\,\delta v^{\rm lsr} \quad \text{with} \quad \delta v^{\rm lsr} = -c\frac{\delta f^{\rm lsr}}{f_{\rm tuned}^{\rm rest}}. \tag{39}$$

But he actually gets

$$v^{\rm lsr}(i) = v_{\rm sys}^{\rm lsr} + (i - i_0)\,\delta v^{\rm obs} \quad \text{with} \quad \delta v^{\rm obs} = -c\frac{\delta f^{\rm obs}}{f_{\rm tuned}^{\rm rest}}. \tag{40}$$

The relative error can be expressed as

$$\frac{\delta v^{\rm obs} - \delta v^{\rm lsr}}{\delta v^{\rm lsr}} = \frac{\delta f^{\rm obs}}{\delta f^{\rm lsr}} - 1 = \frac{1 + d_{\rm sys}^{\rm lsr}}{1 + d_{\rm sys}^{\rm obs}} - 1. \tag{41}$$

We yield to first order in v/c

$$\frac{\delta v^{\rm obs} - \delta v^{\rm lsr}}{\delta v^{\rm lsr}} = \frac{v_{\rm sys}^{\rm obs} - v_{\rm sys}^{\rm lsr}}{c} \sim 10^{-4}. \tag{42}$$

If we tolerate an error of 1/10th of a channel on the velocity, this implies that the velocity scale is correct for about 1000 channels around the systemic velocity. This translates into $\sim 250\,{\rm km\,s^{-1}}$ at 1 mm for a spectral resolution of 195 kHz. This means that the interpretation of the velocity scale of Galactic observations is correct. On the other hand, this could be a problem for extra-galactic observations at extremely high spectral resolution, assuming that the observers do not first downsample their spectra to increase the signal-to-noise ratio per channel...

The easiest way out of the problem would be to write the parameters of the header in the LSR frame in the calibration software that writes the `CLASS` observation with

$$\delta f^{\rm lsr} = \delta f^{\rm obs}\,\frac{1 + d_{\rm sys}^{\rm obs}}{1 + d_{\rm sys}^{\rm lsr}}, \quad \delta v^{\rm lsr} = -c\frac{\delta f^{\rm lsr}}{f_{\rm tuned}^{\rm rest}}, \quad \text{and} \quad d^{\rm lsr} = 1 - \frac{v^{\rm lsr}}{c}. \tag{43}$$

In other words, we trick `CLASS` into thinking that the measurement frame is LSR, while the true measurement frame was the observatory.

4.3 The `MODIFY FREQUENCY` command

This command is used to assign a new rest frequency to the current spectrum. The velocity axis is thus recomputed to set the source systemic velocity at this new rest frequency (see Section 1.4). For consistency the description of the image frequency axis must also be changed.

This command exists because the line rest frequency is sometimes more uncertain than the spectrometer resolution, implying that the line appeared at velocity different from the source systemic velocity. Nowdays,

the bandwidth of the receivers is moreover so large that many lines appear in the same spectrum. For this latter use case, the only way to associate the right velocity to different line rest frequency is to use this command.

4.3.1 Signal frequency axis

The signal frequency axis stays constant under this transform. Only its description changes. This can be written as

$$f_{\rm sig}^{\rm rest}(i) = f_{\rm sig,old}^{\rm rest} + (i - i_{0,\rm old})\,\delta f^{\rm rest}, \tag{44}$$
$$= f_{\rm sig,new}^{\rm rest} + (i - i_{0,\rm new})\,\delta f^{\rm rest}, \tag{45}$$

with

$$\delta f^{\rm rest} = \frac{\delta f_{\rm IF}^{\rm meas}}{1 + d_{\rm sys}^{\rm meas}}. \tag{46}$$

It is then easy to deduce that only the reference channel must be changed as

$$i_{0,\rm new} = i_{0,\rm old} + \frac{f_{\rm sig,new}^{\rm rest} - f_{\rm sig,old}^{\rm rest}}{\delta f_{\rm IF}^{\rm meas}}\left(1 + d_{\rm sys}^{\rm meas}\right) \tag{47}$$

4.3.2 Image frequency axis

The image frequency axis must also be kept fixed. This implies

$$f_{\rm ima}^{\rm rest}(i) = f_{\rm ima,old}^{\rm rest} - (i - i_{0,\rm old})\,\delta f^{\rm rest}, \tag{48}$$
$$= f_{\rm ima,new}^{\rm rest} - (i - i_{0,\rm new})\,\delta f^{\rm rest}, \tag{49}$$

We thus yield for the new tuned signal and image frequency the following relations

$$f_{\rm sig,new}^{\rm rest} = f_{\rm sig,old}^{\rm rest} + (i_{0,\rm new} - i_{0,\rm old})\,\delta f^{\rm rest}, \tag{50}$$
$$f_{\rm ima,new}^{\rm rest} = f_{\rm ima,old}^{\rm rest} - (i_{0,\rm new} - i_{0,\rm old})\,\delta f^{\rm rest}. \tag{51}$$

Adding them gives

$$f_{\rm ima,new}^{\rm rest} + f_{\rm sig,new}^{\rm rest} = f_{\rm sig,old}^{\rm rest} + f_{\rm ima,old}^{\rm rest}. \tag{52}$$

This relation can be used to computes the new image frequency. It can be interpreted as the LO tuning frequency is kept fixed under the transformation. But the side band separation changed.

4.3.3 Velocity axis

The reference channel of the velocity axis was already changed. It only remains to recompute the new velocity resolution associated to the new rest frequency

$$\delta v_{\rm new}^{\rm meas} = -c\frac{\delta f^{\rm meas}}{f_{\rm sig,new}^{\rm rest}}. \tag{53}$$

4.4 The MODIFY VELOCITY command

This command is used to assign a new source systemic velocity to the current spectrum. This command is used when the line rest frequency is known very accurately but the line appears shifted. This implies that the source systemic velocity is slightly offset (see Section 1.3).

4.4.1 Principle

Changing the source systemic velocity is a change of source reference frame. The signal and image frequency scales in the measurement frame must stay constant under this transform. However they change in the source reference frame. Indeed

$$f^{\rm meas} = (1 + d^{\rm meas}_{\rm old})\, f^{\rm rest}_{\rm old} = (1 + d^{\rm meas}_{\rm new})\, f^{\rm rest}_{\rm new}, \tag{54}$$

with

$$d^{\rm meas}_{\rm old} = -\frac{v^{\rm meas}_{\rm sys,old}}{c} \quad \text{and} \quad d^{\rm meas}_{\rm new} = -\frac{v^{\rm meas}_{\rm sys,new}}{c}. \tag{55}$$

Hence

$$f^{\rm rest}_{\rm new} = f^{\rm rest}_{\rm old}\, \frac{1 + d^{\rm meas}_{\rm old}}{1 + d^{\rm meas}_{\rm new}}. \tag{56}$$

4.4.2 The signal frequency axis

The rest signal frequency must be kept fixed. Indeed, the user is trying to characterize a line at the rest signal frequency. But the rest frequency axis changed

$$\begin{aligned} f^{\rm rest}_{\rm sig,new}(i) &= f^{\rm rest}_{\rm sig,tuned} + (i - i_{0,\rm new})\, \delta f^{\rm rest}_{\rm new} && (57)\\ &= \left[f^{\rm rest}_{\rm sig,tuned} + (i - i_{0,\rm old})\, \delta f^{\rm rest}_{\rm old}\right] \frac{1 + d^{\rm meas}_{\rm old}}{1 + d^{\rm meas}_{\rm new}}. && (58) \end{aligned}$$

This equality must be true whatever i. This implies that the reference channel and the frequency resolution must be recomputed with

$$i_{0,\rm new} = i_{0,\rm old} + \frac{f^{\rm rest}_{\rm sig,tuned}}{\delta f^{\rm meas}_{\rm old}}\, \left[d^{\rm meas}_{\rm new} - d^{\rm meas}_{\rm old}\right], \tag{59}$$

and

$$\delta f^{\rm rest}_{\rm new} = \delta f^{\rm rest}_{\rm old}\, \frac{1 + d^{\rm meas}_{\rm old}}{1 + d^{\rm meas}_{\rm new}}, \quad i.e., \quad \delta f^{\rm meas}_{\rm new} = \delta f^{\rm meas}_{\rm old}. \tag{60}$$

The last equation just indicates that the channel spacing, which were measured in the measurement frame, did not changed. Indeed, this is the source frame (and not the measurement frame) which is changed through this transformation. The equation for the computation of the rest frequency resolution from measurement frequency resolution is the usual one (Eq. 5).

4.4.3 The image frequency axis

Now that the reference channel has been computed to keep the rest signal frequency fixed, the only free parameter to compute the new image frequency axis is the tuned image frequency.

$$\begin{aligned} f^{\rm rest}_{\rm ima,new}(i) &= f^{\rm rest}_{\rm ima,tuned,new} - (i - i_{0,\rm new})\, \delta f^{\rm rest}_{\rm new} && (61)\\ &= \left[f^{\rm rest}_{\rm ima,tuned,old} - (i - i_{0,\rm old})\, \delta f^{\rm rest}_{\rm old}\right] \frac{1 + d^{\rm meas}_{\rm old}}{1 + d^{\rm meas}_{\rm new}}. && (62) \end{aligned}$$

$$f^{\rm rest}_{\rm ima,new} = f^{\rm rest}_{\rm ima,old}\, \frac{1 + d^{\rm meas}_{\rm old}}{1 + d^{\rm meas}_{\rm new}} - f^{\rm rest}_{\rm sig,tuned}\, \frac{d^{\rm meas}_{\rm new} - d^{\rm meas}_{\rm old}}{1 + d^{\rm meas}_{\rm new}}. \tag{63}$$

4.4.4 The doppler factor

Note that the systemic velocity is often given in a different frame (LSR, heliocentric) than the measervatory frame. As long as the new systemic velocity is given in the same frame as the old systemic velocity, it is possible to compute the new doppler factor from the old one. Let's assume that the systemic velocity is given in the LSR frame. We have

$$d_{\rm old}^{\rm meas} = -\frac{v_{\rm lsr}^{\rm meas} + v_{\rm sys,old}^{\rm lsr}}{c} \quad \text{and} \quad d_{\rm new}^{\rm meas} = -\frac{v_{\rm lsr}^{\rm meas} + v_{\rm sys,new}^{\rm lsr}}{c}. \tag{64}$$

Subtracting both equation gives

$$d_{\rm new}^{\rm meas} = d_{\rm old}^{\rm meas} + \frac{v_{\rm sys,old}^{\rm lsr} - v_{\rm sys,new}^{\rm lsr}}{c}. \tag{65}$$

5 Velocity conventions

5.1 The generic Doppler formula

The Doppler effect expresses that a source emitting in its rest frame at a frequency $f^{\rm rest}$ will appear to emit at a frequency $f^{\rm obs}$ in the observation frame, which depends on the velocity of the source in the observation frame, $v^{\rm obs}$. The relation between $f^{\rm rest}$ and $f^{\rm obs}$ is given by

$$f^{\rm obs} = f^{\rm rest}\,\frac{\sqrt{c^2 - {v^{\rm obs}}^2}}{c + v_{\parallel}^{\rm obs}}, \tag{66}$$

with ${v^{\rm obs}}^2 = {v_{\parallel}^{\rm obs}}^2 + {v_{\perp}^{\rm obs}}^2$, where $v_{\parallel}^{\rm obs}$ and $v_{\perp}^{\rm obs}$ are respectively the velocity components along and perperdicular to the line of sight. $v_{\parallel}^{\rm obs}$ and $v_{\perp}^{\rm obs}$ are often called the radial and transverse velocity components.

For objects far from the observer, $v_{\parallel}^{\rm obs} \gg v_{\perp}^{\rm obs}$ because of the universe expansion. For close-by objects, Eq. 66 shows that the dependency on $v_{\perp}^{\rm obs}$ is of second order in v/c, while the dependency on $v_{\parallel}^{\rm obs}$ is only of first order in v/c. Hence, for all cases, the Doppler equation is simplified to

$$f^{\rm obs} = f^{\rm rest}\sqrt{\frac{c - v_{\parallel}^{\rm obs}}{c + v_{\parallel}^{\rm obs}}}. \tag{67}$$

In other words, the frequency change under a change of frame is attributed only to the radial velocity between the rest and observation frame.

5.2 The radio velocity convention

If the radial velocity is negligeable compared to the light velocity, we obtain to first order

$$f^{\rm obs} = f^{\rm rest}\left(1 - \frac{v_{\parallel}^{\rm obs}}{c}\right). \tag{68}$$

In this case, we can establish a linear velocity scale (see Sect. 1.4)

$$v^{\rm obs}(i) = v_{\rm sys}^{\rm obs} + (i - i_0)\,\delta v^{\rm obs} \quad \text{with} \quad \delta v^{\rm obs} = -c\frac{\delta f^{\rm obs}}{f_{\rm tuned}^{\rm rest}}. \tag{69}$$

This linear velocity scale is only a first order approximation, which is called the radio velocity convention. Indeed, it is well adapted to the radio spectrometer because 1) their natural output is a spectral axis regularly spaced in frequency and 2) the radio velocity convention gives a linear relation between the velocity and the frequency scale. This is why this convention is the default in CLASS.

5.3 The optical velocity convention

Optical spectrometers (*e.g.*, grates) naturally delivers a spectral axis regularly spaced in wavelength. Eq. 67 can then be rewritten as

$$\lambda^{\rm obs} = \lambda^{\rm rest} \sqrt{\frac{c + v_{\parallel}^{\rm obs}}{c - v_{\parallel}^{\rm obs}}}. \tag{70}$$

The first order approximation gives

$$\lambda^{\rm obs} = \lambda^{\rm rest} \left(1 + \frac{v_{\parallel}^{\rm obs}}{c}\right). \tag{71}$$

In frequency this gives

$$f^{\rm obs} = \frac{f^{\rm rest}}{1 + \frac{v_{\parallel}^{\rm obs}}{c}}. \tag{72}$$

Following the same path as in Sect. 1.4 but now in wavelenght, it is easy to show that we can establish a linear velocity scale proportional to the wavelength axis

$$v_{\rm opt}^{\rm obs}(i) = v_{\rm sys,opt}^{\rm obs} + (i - i_0)\, \delta v_{\rm opt}^{\rm obs} \quad \text{with} \quad \delta v_{\rm opt}^{\rm obs} = +c\, \frac{\delta\lambda^{\rm obs}}{\lambda_{\rm tuned}^{\rm rest}}. \tag{73}$$

This velocity scale is called the optical velocity convention. The relation between this velocity scale and the frequency spectral axis is non-linear. This is why it is not used in CLASS.

5.4 Mixing conventions between observation and analysis

For the moment, CLASS assumes that the data were observed in the observatory frame using a doppler tracked tuning frequency that follows the radio convention. However, the observations may have been done using the relativistic or optical convention. This section deals with the consequences of this mis-match between observation and analysis.

5.4.1 General case

Let's assume that the user wishes to observe a line that appears at the rest frequency, $f_{\rm tuned}^{\rm rest}$, for a source that has a systemic velocity, v_1, in a given velocity convention #1. The rest frame and observatory frame frequencies are linked by the Doppler effect through

$$f_{\rm tuned,conv1}^{\rm obs} = f_{\rm tuned}^{\rm rest}\, D_1(v_1), \tag{74}$$

where $D_1()$ is the Doppler function that can be deduced from Eq. 67, 68, or 72, depending on the assumed convention. For instance, the Doppler function of the optical convention is

$$D_{\rm opt}(v_{\rm opt}) = \frac{1}{1 + \frac{v_{\rm opt}}{c}}. \tag{75}$$

Now, let's assume that the analysis exactly uses the same observed frequency, $f_{\rm tuned,1}^{\rm obs}$, and systemic velocity, v_1, but it interprets it using another Doppler convention and thus Doppler function, $D_2()$. The spectral axis in the rest frame is then computed as

$$f^{\rm rest}(i) = f_{\rm apparent}^{\rm rest} + (i - i_0)\, \delta f_{\rm apparent}^{\rm rest}, \tag{76}$$

with

$$f_{\rm apparent}^{\rm rest} = f_{\rm tuned}^{\rm rest}\, \frac{D_1(v_1)}{D_2(v_1)}, \quad \text{and} \quad \delta f_{\rm apparent}^{\rm rest} = \delta f_{\rm tuned}^{\rm rest}\, \frac{D_1(v_1)}{D_2(v_1)}. \tag{77}$$

This means that the lines will seem to appear at a wrong rest frequency with a wrong full width at half maximum.

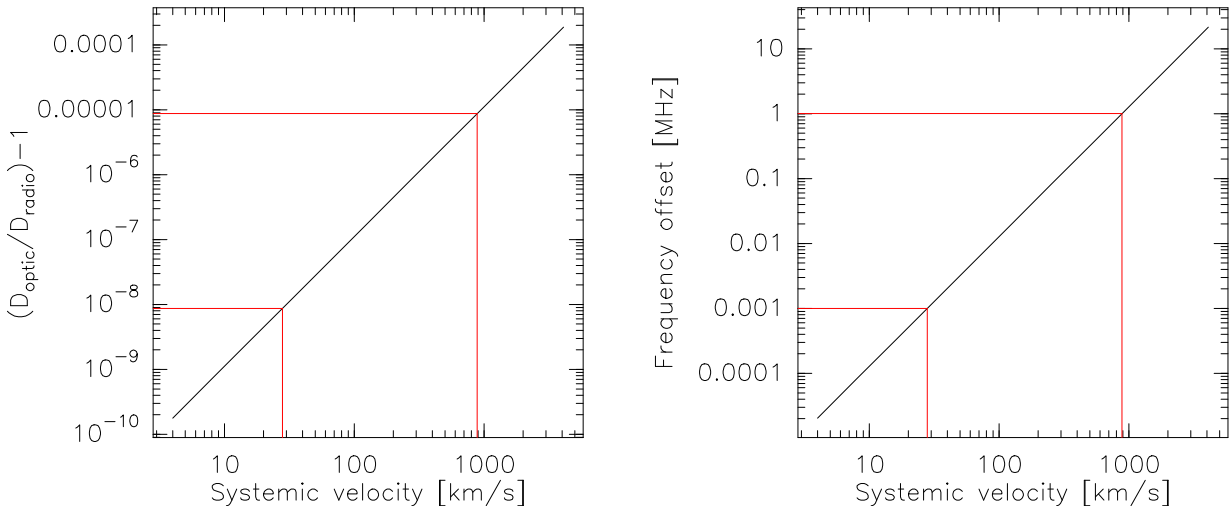

Figure 1: **Left:** Relative error on the dilatation factor. **Right:** Frequency offset for a line at 115 GHz. The red lines correspond to typical LSR velocities for Galactic/extra-Galactic sources, respectively.

5.4.2 The IRAM-30m case as of 2019-07-06

For historical reasons, the velocity convention used to stop the periodic shift due to Earth rotation (see Sect. 3.2) has always been the optical convention at the IRAM-30m. The systemic velocity given by the observer is then interpreted as a radio velocity by `MIRA` and `MRTCAL` to compute the doppler factor from the observatory to the source rest frame. This is exactly the situation described in Sect. 5.4.2. Figure 1 shows the relative error on the dilatation factor (left panel) and the frequency offset for a line at 115 GHz (right panel).

The corresponding frequency offset is 3.0 kHz for a velocity of $28\,\mathrm{km\,s^{-1}}$, and 1.0 MHz for a velocity of $885\,\mathrm{km\,s^{-1}}$. Hence the position of the tuned line for both Galactic and extra-Galactic measurements is affected in a measurable way. Moreover, the dilatation factor differs from 1.0 by a value of the order of 10^{-5} for a velocity of $885\,\mathrm{km\,s^{-1}}$. This means that the frequency of the lines that were not tuned will be wrong by 1/10th of channel when they are located 10 000 channels from the tuned frequency. This is a 2nd order effect that is more difficult to measure with today generation of receivers but it may become measurable with future generations.

To fix this behavior, we first propose to modify the way the spectroscopic section of `CLASS` data is filled by `MRTCAL` as follows

$$f^{\mathrm{rest}}_{\mathrm{sig,new}} = f^{\mathrm{rest}}_{\mathrm{sig,old}} \frac{D_{\mathrm{optic}}(v^{\mathrm{obs}}_{\mathrm{sys,old}})}{D_{\mathrm{radio}}(v^{\mathrm{obs}}_{\mathrm{sys,new}})}, \quad f^{\mathrm{rest}}_{\mathrm{ima,new}} = f^{\mathrm{rest}}_{\mathrm{ima,old}} \frac{D_{\mathrm{optic}}(v^{\mathrm{obs}}_{\mathrm{sys,old}})}{D_{\mathrm{radio}}(v^{\mathrm{obs}}_{\mathrm{sys,new}})}, \tag{78}$$

$$\delta f^{\mathrm{meas}}_{\mathrm{new}} = \delta f^{\mathrm{meas}}_{\mathrm{old}} \frac{D_{\mathrm{optic}}(v^{\mathrm{obs}}_{\mathrm{sys,old}})}{D_{\mathrm{radio}}(v^{\mathrm{obs}}_{\mathrm{sys,new}})}, \quad \text{and} \quad \delta v^{\mathrm{meas}}_{\mathrm{new}} = -c\frac{\delta f^{\mathrm{meas}}_{\mathrm{new}}}{f^{\mathrm{rest}}_{\mathrm{sig,new}}}. \tag{79}$$

The inconvenient of this solution is that it makes $f^{\mathrm{rest}}_{\mathrm{sig,new}}$, $f^{\mathrm{rest}}_{\mathrm{ima,new}}$, and $\delta f^{\mathrm{meas}}_{\mathrm{new}}$ dependent on time because the correction contains the Doppler effect from the observatory to the LSR frame.

A more consistent solution would be that the 30m changes the formula it uses to stop the periodic shift due to Earth rotation and now uses the radio convention.

5.4.3 The APEX case as of 2019-07-06

At APEX, the velocity convention used to stop the periodic shift due to Earth rotation (see Sect. 3.2) is the special relativity one. In order to try to fit into the `CLASS` format, it was decided to deliver the

frequency axis in the source frame at the systemic velocity defined using the special relativity convention and to set the doppler factor to 0. In other words, the interpretation of spectroscopy section of the CLASS header as explained in Table 1 has been tweaked as follows.

`fres` is the channel spacing in the rest frame.

`vres` is the velocity spacing in the frame where the systemic velocity is defined, e.g., LSR.

`voff` is the systemic velocity in the same frame, e.g., LSR.

`doppler` is set to zero.

All other spectroscopic parameters keep their standard definition.

The advantage of this method is that it gives 1) the correct frequency axis in the rest frame, and 2) the correct velocity axis in the velocity frame, as long as the systemic velocity is correct (see Sect. 4 and 4.3). The inconvenient of this method is that the change the systemic velocity as coded in CLASS can not work. Indeed, this change uses the `doppler` parameter (see Sect. 4 and Sect. 4.4), which has been set to zero. While the `MODIFY FREQUENCY` command works as expected, the `MODIFY VELOCITY` command gives unexpected results. Note that the `MODIFY DOPPLER` command will not fix the situation either because it just recomputes the Doppler factor (assuming the radio velocity convention) but it will not change the other parameters of the spectroscopy section accordingly. This command should only be used when the Doppler factor is wrong. This is not the case here as a zero valued Doppler factor is consistent with the definition of the frequency axis.

The only correct solution is to undo the Doppler correction applied at observing time and apply the new Doppler correction associated to the new velocity, all this using the same velocity convention (i.e., the special relativity one) to avoid having CLASS files containing APEX data with different interpretation of the spectroscopy section. We need to update the signal frequency, the image frequency, and the frequency and velocity channel spacing as follows

$$f^{\rm rest}_{\rm sig,new} = f^{\rm rest}_{\rm sig,old}\,\frac{D_{\rm relat}(v^{\rm obs}_{\rm sys,old})}{D_{\rm relat}(v^{\rm obs}_{\rm sys,new})}, \quad f^{\rm rest}_{\rm ima,new} = f^{\rm rest}_{\rm ima,old}\,\frac{D_{\rm relat}(v^{\rm obs}_{\rm sys,old})}{D_{\rm relat}(v^{\rm obs}_{\rm sys,new})}, \tag{80}$$

$$\delta f^{\rm meas}_{\rm new} = \delta f^{\rm meas}_{\rm old}\,\frac{D_{\rm relat}(v^{\rm obs}_{\rm sys,old})}{D_{\rm relat}(v^{\rm obs}_{\rm sys,new})}, \quad \text{and} \quad \delta v^{\rm meas}_{\rm new} = -c\frac{\delta f^{\rm meas}_{\rm new}}{f^{\rm rest}_{\rm sig,new}}. \tag{81}$$

One difficulty is the fact that filling the spectroscopic section as described above implies that the Doppler correction to go from the observatory to the LSR frame has been lost. Fortunately, the velocity composition rule in special relativity implies that

$$\frac{D_{\rm relat}(v^{\rm obs}_{\rm sys,old})}{D_{\rm relat}(v^{\rm obs}_{\rm sys,new})} = \frac{D_{\rm relat}(v^{\rm lsr}_{\rm sys,old})}{D_{\rm relat}(v^{\rm lsr}_{\rm sys,new})}. \tag{82}$$

We can thus replace the previous set of equations by

$$f^{\rm rest}_{\rm sig,new} = f^{\rm rest}_{\rm sig,old}\,\frac{D_{\rm relat}(v^{\rm lsr}_{\rm sys,old})}{D_{\rm relat}(v^{\rm lsr}_{\rm sys,new})}, \quad f^{\rm rest}_{\rm ima,new} = f^{\rm rest}_{\rm ima,old}\,\frac{D_{\rm relat}(v^{\rm lsr}_{\rm sys,old})}{D_{\rm relat}(v^{\rm lsr}_{\rm sys,new})}, \tag{83}$$

$$\delta f^{\rm meas}_{\rm new} = \delta f^{\rm meas}_{\rm old}\,\frac{D_{\rm relat}(v^{\rm lsr}_{\rm sys,old})}{D_{\rm relat}(v^{\rm lsr}_{\rm sys,new})}, \quad \text{and} \quad \delta v^{\rm meas}_{\rm new} = -c\frac{\delta f^{\rm meas}_{\rm new}}{f^{\rm rest}_{\rm sig,new}}. \tag{84}$$

And finally to update the value of $v^{\rm lsr}_{\rm sys}$ to the new one.

This situation should not happen often because galactic targets often have well defined systemic velocity and the general advice is to observe extra-galactic target at the red-shifted frequency to avoid trouble (Gordon et al. 1992).

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