$1,2$

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th

GILDAS
SOFTWARE

$T(i)$

$$f^{\mathrm{obs}}(i) = f^{\mathrm{obs}}_{\mathrm{tuned}} + (i - i_0)\,\delta f^{\mathrm{obs}},$$

$f\mathrm{obs}$

i

$$f_{\text{tuned}}^{\text{obs}}$$

i_0

$$\delta f^{\mathrm{obs}}$$

$$\frac{v^{\mathrm{obs}}}{c} = \frac{f^{\mathrm{rest}} - f^{\mathrm{obs}}}{f^{\mathrm{rest}}} = 1 - \frac{f^{\mathrm{obs}}}{f^{\mathrm{rest}}},$$

v^{obs}

$f\mathrm{rest}$

d^{obs}

$$\frac{f^{\rm obs}}{f^{\rm rest}} = 1 + d^{\rm obs} \quad \text{with} \quad d^{\rm obs} \equiv -\frac{v^{\rm obs}}{c}.$$

$$f^{\mathrm{rest}} = \frac{f^{\mathrm{obs}}}{1 + d^{\mathrm{obs}}}.$$

$$v_{\mathrm{sys}}^{\mathrm{obs}}$$

$$f^{\mathrm{rest}}(i) = f^{\mathrm{rest}}_{\mathrm{tuned}} + (i - i_0)\,\delta f^{\mathrm{rest}} \quad \text{with} \quad \delta f^{\mathrm{rest}} = \frac{\delta f^{\mathrm{obs}}}{1 + d^{\mathrm{obs}}_{\mathrm{sys}}} \quad \text{and} \quad d^{\mathrm{obs}}_{\mathrm{sys}} = -\frac{v^{\mathrm{obs}}_{\mathrm{sys}}}{c}.$$

$$\int \text{rest}$$
$$\text{tuned}$$

$$v_{\mathrm{sys}}^{\mathrm{obs}}$$

$$\frac{f_{\rm tuned}^{\rm obs}}{f_{\rm tuned}^{\rm rest}} = 1 + d_{\rm sys}^{\rm obs} \quad \text{with} \quad d_{\rm sys}^{\rm obs} \equiv -\frac{v_{\rm sys}^{\rm obs}}{c}.$$

$$f^{\mathrm{obs}}(i) - f^{\mathrm{obs}}_{\mathrm{tuned}}$$

$$\delta v^{\mathrm{obs}}(i) \equiv v^{\mathrm{obs}}(i) - v^{\mathrm{obs}}_{\mathrm{sys}}\,.$$

$$\frac{f^{\rm obs}(i)}{f^{\rm rest}_{\rm tuned}} = 1 + d^{\rm obs}(i) \quad \text{with} \quad d^{\rm obs}(i) \equiv -\frac{v^{\rm obs}(i)}{c}.$$

$$\delta v^{\mathrm{obs}}(i) = c\,\frac{f^{\mathrm{obs}}_{\mathrm{tuned}} - f^{\mathrm{obs}}(i)}{f^{\mathrm{rest}}_{\mathrm{tuned}}},$$

$$v^{\mathrm{obs}}(i) = v^{\mathrm{obs}}_{\mathrm{sys}} + (i - i_0)\,\delta v^{\mathrm{obs}} \quad \text{with} \quad \delta v^{\mathrm{obs}} = -c\frac{\delta f^{\mathrm{obs}}}{f^{\mathrm{rest}}_{\mathrm{tuned}}}.$$

$$f_{\mathrm{LO}}^{\mathrm{obs}}$$

$$f_{\mathrm{RF}}^{\mathrm{obs}}$$

$$f_{\mathrm{IF}}^{\mathrm{obs}}$$

$$f_{\mathrm{IF}}^{\mathrm{obs}}(i) = SB_{\mathrm{sign}} \left[f_{\mathrm{RF}}^{\mathrm{obs}}(i) - f_{\mathrm{LO}}^{\mathrm{obs}} \right]$$

$$SB_{\text{sign}} = -1$$

$$SB_{\text{sign}} = +1$$

$$f_{\mathrm{IF}}^{\mathrm{obs}}(i) = f_{\mathrm{IFtuned}}^{\mathrm{obs}} + (i - i_0)\,\delta f_{\mathrm{IF}}^{\mathrm{obs}},$$

$$f_{\mathrm{IFtuned}}^{\mathrm{obs}} \equiv SB_{\mathrm{sign}} \left(f_{\mathrm{RFtuned}}^{\mathrm{obs}} - f_{\mathrm{LO}}^{\mathrm{obs}} \right) \equiv \mathrm{cste.}$$

$$f^{\text{obs}}_{\text{IFtuned}}$$

$$f_{\mathrm{RF}}^{\mathrm{obs}}(i) = (f_{\mathrm{LO}}^{\mathrm{obs}} + SB_{\mathrm{sign}}\, f_{\mathrm{IFtuned}}^{\mathrm{obs}}) + SB_{\mathrm{sign}}\,(i - i_0)\,\delta f_{\mathrm{IF}}^{\mathrm{obs}}.$$

$$f_{\mathrm{RFtuned}}^{\mathrm{obs}}$$

$$f_{\mathrm{sig}}^{\mathrm{obs}}$$

$f_{\text{ima}}^{\text{obs}}$

$$f_{\rm sig}^{\rm obs}(i) = f_{\rm sig,tuned}^{\rm obs} + SB_{\rm sign}\,(i - i_0)\,\delta f_{\rm IF}^{\rm obs} \quad \text{with} \quad f_{\rm sig,tuned}^{\rm obs} \equiv f_{\rm LO}^{\rm obs} + SB_{\rm sign}\, f_{\rm IFtuned}^{\rm obs},$$

$$f_{\rm ima}^{\rm obs}(i) = f_{\rm ima,tuned}^{\rm obs} - SB_{\rm sign}\,(i - i_0)\,\delta f_{\rm IF}^{\rm obs} \quad \text{with} \quad f_{\rm ima,tuned}^{\rm obs} \equiv f_{\rm LO}^{\rm obs} - SB_{\rm sign}\, f_{\rm IFtuned}^{\rm obs}.$$

$$SB_{\mathrm{sign}}\,\delta f_{\mathrm{IF}}^{\mathrm{obs}}$$

$$SB_{\text{sign}}$$

$$\delta f_{\mathrm{IF}}^{\mathrm{obs}}$$

$$\delta f_{\mathrm{IF}}^{\mathrm{obs}} > 0$$

$$f_{\rm sig}^{\rm obs}(i) = f_{\rm sig,tuned}^{\rm obs} + (i - i_0)\,\delta f_{\rm IF}^{\rm obs} \quad \text{with} \quad f_{\rm sig,tuned}^{\rm obs} \equiv f_{\rm LO}^{\rm obs} + SB_{\rm sign}\, f_{\rm IFtuned}^{\rm obs},$$

$$f_{\rm ima}^{\rm obs}(i) = f_{\rm ima,tuned}^{\rm obs} - (i - i_0)\,\delta f_{\rm IF}^{\rm obs} \quad \text{with} \quad f_{\rm ima,tuned}^{\rm obs} \equiv f_{\rm LO}^{\rm obs} - SB_{\rm sign}\, f_{\rm IFtuned}^{\rm obs}.$$

$$\delta f_{\mathrm{IF}}^{\mathrm{obs}} > 0.$$

$$f_{\mathrm{sig}}^{\mathrm{rest}}(i) = f_{\mathrm{sig,tuned}}^{\mathrm{rest}} + (i - i_0)\,\delta f^{\mathrm{rest}},$$

$$f_{\mathrm{ima}}^{\mathrm{rest}}(i) = f_{\mathrm{ima,tuned}}^{\mathrm{rest}} - (i - i_0)\,\delta f^{\mathrm{rest}},$$

$$\delta f^{\mathrm{rest}} = \frac{\delta f_{\mathrm{IF}}^{\mathrm{obs}}}{1 + d_{\mathrm{sys}}^{\mathrm{obs}}} \quad \text{and} \quad d_{\mathrm{sys}}^{\mathrm{obs}} = -\frac{v_{\mathrm{sys}}^{\mathrm{obs}}}{c}.$$

$$f_{\text{sig,tuned}}^{\text{rest}}$$

$$f_{\text{ima,tuned}}^{\text{rest}}$$

$$SB_{\rm sep}^{\rm obs} \equiv SB_{\rm sign} \left[f_{\rm sig,tuned}^{\rm obs} - f_{\rm ima,tuned}^{\rm obs} \right] .$$

$$SB_{\mathrm{sep}}^{\mathrm{obs}} = \mathrm{cste} = 2\, f_{\mathrm{IFtuned}}^{\mathrm{obs}}.$$

$$\frac{f^{\text{obs}}_{\text{sig,tuned}}}{f^{\text{rest}}_{\text{sig,tuned}}} = 1 + d^{\text{obs}}_{\text{sys}} \qquad \frac{f^{\text{obs}}_{\text{ima,tuned}}}{f^{\text{rest}}_{\text{ima,tuned}}} = 1 + d^{\text{obs}}_{\text{sys}}.$$

$$SB_{\mathrm{sep}}^{\mathrm{rest}} \equiv SB_{\mathrm{sign}} \left[f_{\mathrm{sig,tuned}}^{\mathrm{rest}} - f_{\mathrm{ima,tuned}}^{\mathrm{rest}} \right] .$$

$$SB_{\mathrm{sep}}^{\mathrm{obs}} = SB_{\mathrm{sep}}^{\mathrm{rest}} \ \left[1 + d_{\mathrm{sys}}^{\mathrm{obs}}\right],$$

$$f_{\mathrm{ima,tuned}}^{\mathrm{rest}} = f_{\mathrm{sig,tuned}}^{\mathrm{rest}} - \frac{SB_{\mathrm{sep}}^{\mathrm{obs}}}{SB_{\mathrm{sign}}\left[1 + d_{\mathrm{sys}}^{\mathrm{obs}}\right]}.$$

$$f^{\rm lsr} = f^{\rm obs} + f^{\rm rest}\,\frac{v_{\rm sys}^{\rm obs} - v_{\rm sys}^{\rm lsr}}{c},$$

$$f^{\mathrm{obs}} = f^{\mathrm{rest}} \left(1 - \frac{v_{\mathrm{sys}}^{\mathrm{obs}}}{c}\right) \quad \text{and} \quad f^{\mathrm{lsr}} = f^{\mathrm{rest}} \left(1 - \frac{v_{\mathrm{sys}}^{\mathrm{lsr}}}{c}\right).$$

$$\left[SB_{\rm sign}\, f^{\rm rest}\, (v_{\rm sys}^{\rm obs} - v_{\rm sys}^{\rm lsr})/c\right]$$

$$f_{\mathrm{sig,tuned}}^{\mathrm{lsr}} = f_{\mathrm{sig,tuned}}^{\mathrm{obs}} + f_{\mathrm{sig,tuned}}^{\mathrm{rest}} \frac{v_{\mathrm{sys}}^{\mathrm{obs}} - v_{\mathrm{sys}}^{\mathrm{lsr}}}{c}.$$

$$f^{\mathrm{lsr}}(i) = f^{\mathrm{obs}}(i) + f^{\mathrm{rest}}_{\mathrm{sig,tuned}} \, \frac{v^{\mathrm{obs}}_{\mathrm{sys}} - v^{\mathrm{lsr}}_{\mathrm{sys}}}{c},$$

$$f^{\mathrm{lsr}}(i) = f^{\mathrm{obs}}(i) + f^{\mathrm{rest}}(i)\,\frac{v_{\mathrm{sys}}^{\mathrm{obs}} - v_{\mathrm{sys}}^{\mathrm{lsr}}}{c}.$$

$$\Delta f^{\mathrm{lsr}}(i) = (i - i_0)\, \delta f^{\mathrm{rest}}\, \frac{v_{\mathrm{sys}}^{\mathrm{obs}} - v_{\mathrm{sys}}^{\mathrm{lsr}}}{c}.$$

$$i = i_0$$

$$f_{\text{tuned,sig}}^{\text{rest}}$$

$f_{\text{tuned,ima}}^{\text{rest}}$

$$\delta f^{\mathrm{meas}}$$

$\mathrm{km\,s^{-1}}$

$$\delta v^{\mathrm{meas}}$$

$$v_{\text{sys}}^{\text{meas}}$$

$$d_{\mathrm{sys}}^{\mathrm{meas}}$$

$$f_{\mathrm{sig}}^{\mathrm{rest}}(i) = f_{\mathrm{sig,tuned}}^{\mathrm{rest}} + (i - i_0)\,\delta f^{\mathrm{rest}},$$

$$f_{\mathrm{ima}}^{\mathrm{rest}}(i) = f_{\mathrm{ima,tuned}}^{\mathrm{rest}} - (i - i_0)\,\delta f^{\mathrm{rest}},$$

$$\delta f^{\mathrm{rest}} = \frac{\delta f_{\mathrm{IF}}^{\mathrm{meas}}}{1 + d_{\mathrm{sys}}^{\mathrm{meas}}} \quad \text{and} \quad d_{\mathrm{sys}}^{\mathrm{meas}} = -\frac{v_{\mathrm{sys}}^{\mathrm{meas}}}{c},$$

$$\delta f_{\mathrm{IF}}^{\mathrm{meas}}$$

$$v_{\text{sys}}^{\text{meas}}$$

$$v^{\mathrm{meas}}(i) = v^{\mathrm{meas}}_{\mathrm{sys}} + (i - i_0)\,\delta v^{\mathrm{meas}} \quad \text{with} \quad \delta v^{\mathrm{meas}} = -c\frac{\delta f^{\mathrm{meas}}}{f^{\mathrm{rest}}_{\mathrm{tuned}}}.$$

$$v^{\mathrm{lsr}}(i) = v^{\mathrm{lsr}}_{\mathrm{sys}} + (i - i_0)\,\delta v^{\mathrm{lsr}} \quad \text{with} \quad \delta v^{\mathrm{lsr}} = -c\frac{\delta f^{\mathrm{lsr}}}{f^{\mathrm{rest}}_{\mathrm{tuned}}}.$$

$$v^{\mathrm{lsr}}(i) = v^{\mathrm{lsr}}_{\mathrm{sys}} + (i - i_0)\,\delta v^{\mathrm{obs}} \quad \text{with} \quad \delta v^{\mathrm{obs}} = -c\frac{\delta f^{\mathrm{obs}}}{f^{\mathrm{rest}}_{\mathrm{tuned}}}.$$

$$\frac{\delta v^{\rm obs} - \delta v^{\rm lsr}}{\delta v^{\rm lsr}} = \frac{\delta f^{\rm obs}}{\delta f^{\rm lsr}} - 1 = \frac{1 + d_{\rm sys}^{\rm lsr}}{1 + d_{\rm sys}^{\rm obs}} - 1.$$

v/c

$$\frac{\delta v^{\rm obs} - \delta v^{\rm lsr}}{\delta v^{\rm lsr}} = \frac{v_{\rm sys}^{\rm obs} - v_{\rm sys}^{\rm lsr}}{c} \sim 10^{-4}.$$

$\sim 250\,\mathrm{km\,s^{-1}}$

mm

kHz

$$\delta f^{\mathrm{lsr}} = \delta f^{\mathrm{obs}} \, \frac{1 + d_{\mathrm{sys}}^{\mathrm{obs}}}{1 + d_{\mathrm{sys}}^{\mathrm{lsr}}}, \quad \delta v^{\mathrm{lsr}} = -c \frac{\delta f^{\mathrm{lsr}}}{f_{\mathrm{tuned}}^{\mathrm{rest}}}, \quad \text{and} \quad d^{\mathrm{lsr}} = 1 - \frac{v^{\mathrm{lsr}}}{c}.$$

$$f_{\mathrm{sig}}^{\mathrm{rest}}(i)$$

$=$

$$f_{\mathrm{sig,old}}^{\mathrm{rest}} + (i - i_{0,\mathrm{old}})\, \delta f^{\mathrm{rest}},$$

$$f^{\mathrm{rest}}_{\mathrm{sig,new}} + (i - i_{0,\mathrm{new}})\, \delta f^{\mathrm{rest}},$$

$$\delta f^{\mathrm{rest}} = \frac{\delta f_{\mathrm{IF}}^{\mathrm{meas}}}{1 + d_{\mathrm{sys}}^{\mathrm{meas}}}.$$

$$i_{0,\mathrm{new}} = i_{0,\mathrm{old}} + \frac{f_{\mathrm{sig,new}}^{\mathrm{rest}} - f_{\mathrm{sig,old}}^{\mathrm{rest}}}{\delta f_{\mathrm{IF}}^{\mathrm{meas}}} \left(1 + d_{\mathrm{sys}}^{\mathrm{meas}}\right)$$

$$f_{\text{ima}}^{\text{rest}}(i)$$

$$f^{\mathrm{rest}}_{\mathrm{ima,old}} - \left(i - i_{0,\mathrm{old}}\right) \delta f^{\mathrm{rest}},$$

$$f_{\mathrm{ima,new}}^{\mathrm{rest}} - \left(i - i_{0,\mathrm{new}}\right) \delta f^{\mathrm{rest}},$$

$$f_{\text{sig,new}}^{\text{rest}}$$

$$f_{\mathrm{sig,old}}^{\mathrm{rest}} + \left(i_{0,\mathrm{new}} - i_{0,\mathrm{old}}\right) \delta f^{\mathrm{rest}},$$

$$f_{\text{ima,new}}^{\text{rest}}$$

$$f^{\mathrm{rest}}_{\mathrm{ima,old}} - \left(i_{0,\mathrm{new}} - i_{0,\mathrm{old}}\right) \delta f^{\mathrm{rest}}.$$

$$f^{\text{rest}}_{\text{ima,new}} + f^{\text{rest}}_{\text{sig,new}} = f^{\text{rest}}_{\text{sig,old}} + f^{\text{rest}}_{\text{ima,old}}.$$

$$\delta v_{\mathrm{new}}^{\mathrm{meas}} = -c \frac{\delta f^{\mathrm{meas}}}{f_{\mathrm{sig,new}}^{\mathrm{rest}}}.$$

$$f^{\mathrm{meas}} = (1 + d_{\mathrm{old}}^{\mathrm{meas}})\, f_{\mathrm{old}}^{\mathrm{rest}} = (1 + d_{\mathrm{new}}^{\mathrm{meas}})\, f_{\mathrm{new}}^{\mathrm{rest}},$$

$$d_{\text{old}}^{\text{meas}} = -\frac{v_{\text{sys,old}}^{\text{meas}}}{c} \quad \text{and} \quad d_{\text{new}}^{\text{meas}} = -\frac{v_{\text{sys,new}}^{\text{meas}}}{c}.$$

$$f_{\mathrm{new}}^{\mathrm{rest}} = f_{\mathrm{old}}^{\mathrm{rest}} \, \frac{1 + d_{\mathrm{old}}^{\mathrm{meas}}}{1 + d_{\mathrm{new}}^{\mathrm{meas}}}.$$

$$f_{\mathrm{sig,new}}^{\mathrm{rest}}(i)$$

$$f_{\mathrm{sig,tuned}}^{\mathrm{rest}} + (i - i_{0,\mathrm{new}})\, \delta f_{\mathrm{new}}^{\mathrm{rest}}$$

$$\left[f^{\mathrm{rest}}_{\mathrm{sig,tuned}} + (i - i_{0,\mathrm{old}})\,\delta f^{\mathrm{rest}}_{\mathrm{old}}\right]\,\frac{1 + d^{\mathrm{meas}}_{\mathrm{old}}}{1 + d^{\mathrm{meas}}_{\mathrm{new}}}.$$

$$i_{0,\mathrm{new}} = i_{0,\mathrm{old}} + \frac{f^{\mathrm{rest}}_{\mathrm{sig,tuned}}}{\delta f^{\mathrm{meas}}_{\mathrm{old}}} \left[d^{\mathrm{meas}}_{\mathrm{new}} - d^{\mathrm{meas}}_{\mathrm{old}} \right],$$

$$\delta f_{\rm new}^{\rm rest} = \delta f_{\rm old}^{\rm rest}\,\frac{1+d_{\rm old}^{\rm meas}}{1+d_{\rm new}^{\rm meas}}, \quad i.e., \quad \delta f_{\rm new}^{\rm meas} = \delta f_{\rm old}^{\rm meas}.$$

$$f_{\text{ima,new}}^{\text{rest}}(i)$$

$$f_{\mathrm{ima,tuned,new}}^{\mathrm{rest}} - \left(i - i_{0,\mathrm{new}}\right) \delta f_{\mathrm{new}}^{\mathrm{rest}}$$

$$\left[f^{\mathrm{rest}}_{\mathrm{ima,tuned,old}} - \left(i - i_{0,\mathrm{old}}\right) \delta f^{\mathrm{rest}}_{\mathrm{old}}\right] \; \frac{1 + d^{\mathrm{meas}}_{\mathrm{old}}}{1 + d^{\mathrm{meas}}_{\mathrm{new}}} .$$

$$f^{\mathrm{rest}}_{\mathrm{ima,new}} = f^{\mathrm{rest}}_{\mathrm{ima,old}} \frac{1 + d^{\mathrm{meas}}_{\mathrm{old}}}{1 + d^{\mathrm{meas}}_{\mathrm{new}}} - f^{\mathrm{rest}}_{\mathrm{sig,tuned}} \frac{d^{\mathrm{meas}}_{\mathrm{new}} - d^{\mathrm{meas}}_{\mathrm{old}}}{1 + d^{\mathrm{meas}}_{\mathrm{new}}}.$$

$$d_{\rm old}^{\rm meas} = -\frac{v_{\rm lsr}^{\rm meas} + v_{\rm sys,old}^{\rm lsr}}{c} \quad \text{and} \quad d_{\rm new}^{\rm meas} = -\frac{v_{\rm lsr}^{\rm meas} + v_{\rm sys,new}^{\rm lsr}}{c}.$$

$$d_{\rm new}^{\rm meas} = d_{\rm old}^{\rm meas} + \frac{v_{\rm sys,old}^{\rm lsr} - v_{\rm sys,new}^{\rm lsr}}{c}.$$

$$f^{\mathrm{obs}} = f^{\mathrm{rest}} \frac{\sqrt{c^2 - {v^{\mathrm{obs}}}^2}}{c + v_{\parallel}^{\mathrm{obs}}},$$

$$v^{\mathrm{obs}\,2} = v_{\parallel}^{\mathrm{obs}\,2} + v_{\perp}^{\mathrm{obs}\,2}$$

$$v_{\parallel}^{\mathrm{obs}}$$

$$v_{\perp}^{\mathrm{obs}}$$

$$v_{\parallel}^{\mathrm{obs}} \gg v_{\perp}^{\mathrm{obs}}$$

$$f^{\mathrm{obs}} = f^{\mathrm{rest}} \sqrt{\frac{c - v_{\parallel}^{\mathrm{obs}}}{c + v_{\parallel}^{\mathrm{obs}}}}.$$

$$f^{\mathrm{obs}} = f^{\mathrm{rest}} \left(1 - \frac{v_{\parallel}^{\mathrm{obs}}}{c} \right) .$$

$$v^{\mathrm{obs}}(i) = v^{\mathrm{obs}}_{\mathrm{sys}} + (i - i_0)\,\delta v^{\mathrm{obs}} \quad \text{with} \quad \delta v^{\mathrm{obs}} = -c\frac{\delta f^{\mathrm{obs}}}{f^{\mathrm{rest}}_{\mathrm{tuned}}}.$$

$$\lambda^{\mathrm{obs}} = \lambda^{\mathrm{rest}} \sqrt{\frac{c + v_{\parallel}^{\mathrm{obs}}}{c - v_{\parallel}^{\mathrm{obs}}}}.$$

$$\lambda^{\mathrm{obs}} = \lambda^{\mathrm{rest}} \left(1 + \frac{v_{\parallel}^{\mathrm{obs}}}{c} \right).$$

$$f^{\mathrm{obs}} = \frac{f^{\mathrm{rest}}}{1 + \frac{v_{\parallel}^{\mathrm{obs}}}{c}}.$$

$$v_{\rm opt}^{\rm obs}(i) = v_{\rm sys,opt}^{\rm obs} + (i - i_0)\,\delta v_{\rm opt}^{\rm obs} \quad \text{with} \quad \delta v_{\rm opt}^{\rm obs} = +c\,\frac{\delta\lambda^{\rm obs}}{\lambda_{\rm tuned}^{\rm rest}}.$$

v_1

$$f^{\text{obs}}_{\text{tuned,conv1}} = f^{\text{rest}}_{\text{tuned}}\, D_1(v_1),$$

$$D_1()$$

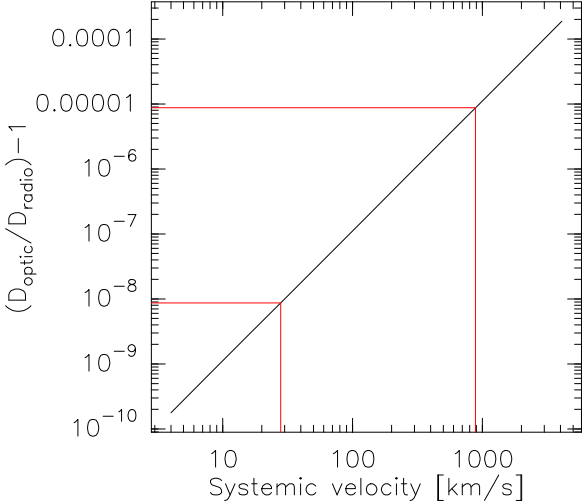
$$D_{\mathrm{opt}}(v_{\mathrm{opt}}) = \frac{1}{1 + \frac{v_{\mathrm{opt}}}{c}}.$$

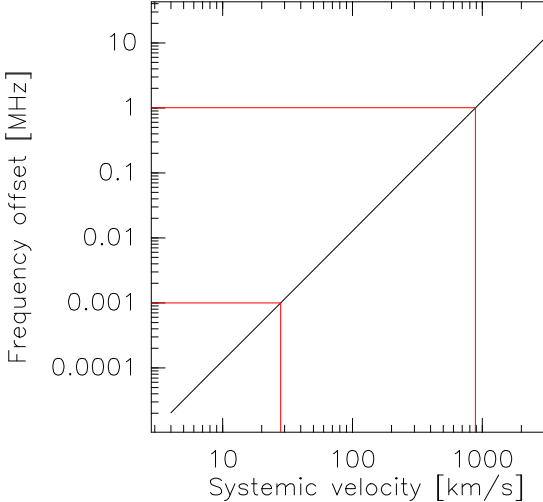
$$f_{\text{tuned},1}^{\text{obs}}$$

$D_2()$

$$f^{\mathrm{rest}}(i) = f^{\mathrm{rest}}_{\mathrm{apparent}} + (i - i_0)\,\delta f^{\mathrm{rest}}_{\mathrm{apparent}},$$

$$f_{\text{apparent}}^{\text{rest}} = f_{\text{tuned}}^{\text{rest}} \frac{D_1(v_1)}{D_2(v_1)}, \quad \text{and} \quad \delta f_{\text{apparent}}^{\text{rest}} = \delta f_{\text{tuned}}^{\text{rest}} \frac{D_1(v_1)}{D_2(v_1)}.$$


0.0001
0.00001
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-10}
$(D_{optic}/D_{radio})-1$
10
100
1000
Systemic velocity [km/s]


10
1
0.1
0.01
0.001
0.0001
Frequency offset [MHz]
10
100
1000
Systemic velocity [km/s]

10^{-5}

$$f^{\rm rest}_{\rm sig,new} = f^{\rm rest}_{\rm sig,old}\,\frac{D_{\rm optic}(v^{\rm obs}_{\rm sys,old})}{D_{\rm radio}(v^{\rm obs}_{\rm sys,new})}, \quad f^{\rm rest}_{\rm ima,new} = f^{\rm rest}_{\rm ima,old}\,\frac{D_{\rm optic}(v^{\rm obs}_{\rm sys,old})}{D_{\rm radio}(v^{\rm obs}_{\rm sys,new})},$$

$$\delta f_{\rm new}^{\rm meas} = \delta f_{\rm old}^{\rm meas}\,\frac{D_{\rm optic}(v_{\rm sys,old}^{\rm obs})}{D_{\rm radio}(v_{\rm sys,new}^{\rm obs})}, \quad \text{and} \quad \delta v_{\rm new}^{\rm meas} = -c\frac{\delta f_{\rm new}^{\rm meas}}{f_{\rm sig,new}^{\rm rest}}.$$

$$f^{\text{rest}}_{\text{sig,new}}$$

$$f_{\text{ima,new}}^{\text{rest}}$$

$$\delta f_{\text{new}}^{\text{meas}}$$

$$f^{\mathrm{rest}}_{\mathrm{sig,new}} = f^{\mathrm{rest}}_{\mathrm{sig,old}} \frac{D_{\mathrm{relat}}(v^{\mathrm{obs}}_{\mathrm{sys,old}})}{D_{\mathrm{relat}}(v^{\mathrm{obs}}_{\mathrm{sys,new}})}, \quad f^{\mathrm{rest}}_{\mathrm{ima,new}} = f^{\mathrm{rest}}_{\mathrm{ima,old}} \frac{D_{\mathrm{relat}}(v^{\mathrm{obs}}_{\mathrm{sys,old}})}{D_{\mathrm{relat}}(v^{\mathrm{obs}}_{\mathrm{sys,new}})},$$

$$\delta f_{\rm new}^{\rm meas} = \delta f_{\rm old}^{\rm meas}\,\frac{D_{\rm relat}(v_{\rm sys,old}^{\rm obs})}{D_{\rm relat}(v_{\rm sys,new}^{\rm obs})}, \quad \text{and} \quad \delta v_{\rm new}^{\rm meas} = -c\,\frac{\delta f_{\rm new}^{\rm meas}}{f_{\rm sig,new}^{\rm rest}}.$$

$$\frac{D_{\mathrm{relat}}(v^{\mathrm{obs}}_{\mathrm{sys,old}})}{D_{\mathrm{relat}}(v^{\mathrm{obs}}_{\mathrm{sys,new}})} = \frac{D_{\mathrm{relat}}(v^{\mathrm{lsr}}_{\mathrm{sys,old}})}{D_{\mathrm{relat}}(v^{\mathrm{lsr}}_{\mathrm{sys,new}})}.$$

$$f^{\rm rest}_{\rm sig,new} = f^{\rm rest}_{\rm sig,old}\,\frac{D_{\rm relat}(v^{\rm lsr}_{\rm sys,old})}{D_{\rm relat}(v^{\rm lsr}_{\rm sys,new})}, \quad f^{\rm rest}_{\rm ima,new} = f^{\rm rest}_{\rm ima,old}\,\frac{D_{\rm relat}(v^{\rm lsr}_{\rm sys,old})}{D_{\rm relat}(v^{\rm lsr}_{\rm sys,new})},$$

$$\delta f_{\rm new}^{\rm meas} = \delta f_{\rm old}^{\rm meas}\,\frac{D_{\rm relat}(v_{\rm sys,old}^{\rm lsr})}{D_{\rm relat}(v_{\rm sys,new}^{\rm lsr})}, \quad \text{and} \quad \delta v_{\rm new}^{\rm meas} = -c\,\frac{\delta f_{\rm new}^{\rm meas}}{f_{\rm sig,new}^{\rm rest}}.$$

$$v_{\rm sys}^{\rm lsr}$$