


# IRAM Memo 2021-?
# Weighting scheme in **CLASS**

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**Abstract**

The weighting scheme of **CLASS** was slightly inconsistent when summing or averaging spectra observed in different observing modes (frequency switching, wobbler switching, and position switching). A first attempt at solving this problem that was implemented on Sep. 17th, 2019, led to incorrect system temperatures in the output spectrum when averaging or summing folded frequency switched spectra. This in turn brake the associativity of the average or sum commands: When averaging three folded frequency switched spectra S1, S2, and S3, the noise level of (S1+S2+S3) was slightly different from the noise level of ((S1+S2)+S3)).

In this memo, we describe the problem and propose a solution that was implemented in the mar21 release of **GILDAS**. We ask all our users that currently use a version of **GILDAS** between oct19 and feb21 to update their version to the newest one.

Keywords: **CLASS** Data Format

Related documents: Averaging spectra with **CLASS** (memo 2009-4), **MRTCAL** user manual.

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# 1 Weighting modes

In **CLASS**, the `SET WEIGHT` command offers 3 weighting modes (see IRAM memo 2009-4):

- `EQUAL`: the weights are set to $w_{\rm E} = 1$,
- `TIME`: the weights are set to $w_{\rm T} = t \times f_{\rm res}/T_{\rm sys}^2$, where $t$ is the integration time in seconds, $f_{\rm res}$ the frequency resolution in megaHertz, and $T_{\rm sys}$ the system temperature in Kelvin.
- `SIGMA`: the weights are set to $w_{\rm S} = 10^{-6}/\sigma^2$, where $\sigma$ is the rms noise in Kelvin. $\sigma$ is usually precomputed by the `BASELINE` on channel ranges with no signal. The $10^{-6}$ is introduced to match the s.MHz/K$^2$ unit of the `TIME` weight.

These are relative weights between spectra to be combined in commands like `AVERAGE`, `XY_MAP`, etc. Absolute weights involve an additional constant factor which is not described here. The weighting modes can not be mixed: when combining spectra, all weights are `EQUAL`, or all weights are `TIME`, or all weights are `SIGMA`. Note that the `TIME` weight is a theoretical value based on parameter values taken at observation time, while the `SIGMA` is a measured value based on the actual noise property of the spectrum.

# 2 Switching modes

**CLASS** supports 3 major switching modes (see the **MRTCAL** user documentation for a more complete description):

- position switching (abbreviated into psw),
- wobbler switching (abbreviated into wsw),
- frequency switching (abbreviated into fsw).

The frequency switching mode can be divided into 2 subcategories:

- unfolded frequency switching (abbreviated ufsw),
- folded frequency switching (abbreviated ffsw).

Up to sep19 **GILDAS** version, **CLASS** used the same weighting definitions for all switching modes. In particular for the `TIME` weights

$$w_{\rm T,psw} = w_{\rm T,wsw} = w_{\rm T,ufsw} = w_{\rm T,ffsw} = w_{\rm T}, \tag{1}$$

where $w_{\rm T}$ is defined above.

# 3 Fixing folded frequency switching weight

Looking closer to frequency switching, the folding operation folds a spectrum on itself to shift and average the two phases into a single line. This operation decreases the noise level by a factor $\sqrt{2}$ at the frequency of the line. Moreover, when doing this, **CLASS** keeps constant key observing parameters like the observing time to remain consistent with the actual observing run at the telescope. From Eq. 1, this means that an unfolded frequency switching spectrum and its folded version wrongly had the same theoretical noises (and, thus, the same `TIME` weights), while they correctly had different measured noise (and different `SIGMA` weights).

This inconsistency between unfolded fsw and folded fsw spectra was in general OK, as **CLASS** commands refuse to combine these two kinds of spectra. However, it is possible to combine (average, sum, ...) folded fsw spectra with psw and wsw. As the theoretical folded fsw noise does not reflect the actual

noise, the relative TIME weights were incorrect and the spectra from different switching modes were not combined properly.

This problem was fixed on Sep. 17th, 2019, and the patch was first included in the oct19 **GILDAS** release. Starting from that date, folded frequency switching spectra were given an additional factor 2 to their TIME weight:

$$w_{\mathrm{T,ffsw}} = 2 \times w_{\mathrm{T}} \tag{2}$$

This has no effect when mixing folded fsw spectra altogether (as all the relative weights include the same extra factor), but this fixes the combination of folded fsw with other allowed switching modes.

# 4 Fixing the AVERAGE output system temperature

The commands AVERAGE, ACCUMULATE, and STITCH combine the spectra available in the current index according to their relative weights, and produce a single output spectrum with a proper description in terms of observing time, resolution, and system temperature.

Assuming the simplest case of a set of $N$ spectra with identical observing time, resolution, and system temperature, the output spectrum has a resolution identical to the input one, and an observing time which is the sum of all the inputs one. The output system temperature is then derived by reverting the theoritical time weight formula

$$w_{\mathrm{out}} = \sum_N w_{\mathrm{in}} \tag{3}$$

$$T_{\mathrm{sys,out}} = w_T^{-1}(w_{\mathrm{out}}, f_{\mathrm{res,out}}, t_{\mathrm{out}}) \tag{4}$$

When averaging folded fsw with TIME weighting, this gives:

$$w_{\mathrm{T,out}} = \sum_N w_{\mathrm{T,ffsw}} = N \times 2 \times t_{\mathrm{in}} \times f_{\mathrm{res}}/T^2_{\mathrm{sys,in}} = 2 \times t_{\mathrm{out}} \times f_{\mathrm{res}}/T^2_{\mathrm{sys,in}} \tag{5}$$

If we reverse the generic $w_{\mathrm{T}}$ formula, this gives an unexpected change of system temperature:

$$T_{\mathrm{sys,out}} = \frac{T_{\mathrm{sys,in}}}{\sqrt{2}} \tag{6}$$

In other words, when the factor 2 was introduced in 2019, it was not added symetrically in the $w_T^{-1}$ reverse operation done by AVERAGE. The result was that when all the averaged spectra were folded fsw, the output Tsys was underestimated by a factor $\sqrt{2}$. As of 24-feb-2021 and release mar21, this is fixed now:

$$w^{-1}_{\mathrm{T,ffsw}} = \frac{1}{2} \times w^{-1}_{\mathrm{T}} \tag{7}$$

This reverse function is used when mixing folded fsw spectra only. When mixing psw or wsw spectra only, the generic $w_T^{-1}$ function is used. Finally, when mixing folded fsw with psw or wsw, the output Tsys is more complicated to evaluate, as there is no unique weight formula to revert. The chosen solution is to use also the generic $w_{\mathrm{T}}^{-1}$ function, leading to an approximate evaluation of the output system temperature. In this latter case, we also introduce a *mix switching mode* (reflecting the combination of spectra observed in different modes), and the associated section in the spectrum header is emptied except for this code.

For example, when mixing a one psw or wsw spectrum with one fsw spectrum of same system temperature, integration time, and frequency resolution, this gives:

$$w_{\mathrm{T,psw}} = t_{\mathrm{in}} \times f_{\mathrm{res}}/T^2_{\mathrm{sys,in}} \tag{8}$$

$$w_{\mathrm{T,ffsw}} = 2 \times t_{\mathrm{in}} \times f_{\mathrm{res}}/T^2_{\mathrm{sys,in}} \tag{9}$$

$$w_{\mathrm{T,mix}} = t_{\mathrm{out}} \times f_{\mathrm{res}}/T^2_{\mathrm{sys,mix}} \tag{10}$$

with $t_{\rm out} = 2 \times t_{\rm in}$ (sum of input integration times). Since $w_{\rm T,mix} = w_{\rm T,psw} + w_{\rm T,ffsw}$, this yields:

$$T_{\rm sys,mix} = \sqrt{\frac{t_{\rm out} \times f_{\rm res}}{w_{\rm T,psw} + w_{\rm T,ffsw}}} = \sqrt{\frac{2 \times t_{\rm in}}{3 \times t_{\rm in}/T_{\rm sys,in}^2}} = T_{\rm sys,in} \times \sqrt{\frac{2}{3}} \tag{11}$$

We see that the resulting system temperature is underestimated because the reverse weight formula is not ideal in the mixed case.

# 5 Consequences

For all versions of **GILDAS** between oct19 and feb21

- Summing, averaging, or stitching spectra whose observing mode is either position switching, wobbler switching, or *unfolded* frequency switching gives the expected output spectrum.
- Summing, averaging, or stitching *folded* frequency switching spectra could lead to slightly incorrect noise estimations in some specific conditions.
  - If the user is summing, averaging, or stitching all the spectra in a single command, the output spectrum intensities and their associated noise level are behaving as expected.
  - Only when the user is summing, averaging, or stitching *folded* frequency switching spectra in subsequent calls to the `AVERAGE`, `ACCUMULATE`, or `STITCH` commands would lead to incorrect noise levels. The shape of the line is nevertheless preserved.

# 6 Test data

We test here the `AVERAGE` command behavior and the noise properties on a frequency switch dataset. Here we use the 3 frequency switch tracked spectra observed on 03-jun-2013, scans 100, 101, and 102, on the Vespa 4 backend part (`30ME2VLI-V04`).

Table 1: Status of the raw spectra w/t their theoretical and measured noise in all **CLASS** versions. $t$ is the integration time, $f_{\rm res}$ is the frequency resolution. The theoretical noise is computed from $T_{\rm sys}$, $t$, and $f_{\rm res}$ (in Hz): $\sigma = T_{\rm sys}/\sqrt{t \times f_{\rm res}}$. The measured noise is computed by the `BASELINE` command, excluding the windows with signal.

| Scan | Tsys (K) | t (s) | $f_{\rm res}$ (MHz) | Theoretical noise (K) | Measured noise (K) | Status |
|---|---|---|---|---|---|---|
| 100 | 224.66 | 56.61 | 0.0195 | 0.214 | 0.219 | OK |
| 101 | 224.53 | 56.61 | 0.0195 | 0.214 | 0.220 | OK |
| 102 | 224.40 | 56.61 | 0.0195 | 0.213 | 0.217 | OK |

Table 2: Same as Table 1, for the averaged spectra produced by the command `AVERAGE` in **CLASS** version feb21. We combine the 3 spectra either in a single call (100+101+102), or in 2 calls with an intermediate average (100+101, then (100+101)+102). This **CLASS** version gives unexpected results (in red). 1) After a single average, the system temperature, and thus the theoretical noise, is underestimated by a factor $\sqrt{2}$, but the measured noise is correct as the weighted average used proper weights. 2) when averaging in several calls, the error on the system temperature propagates also to the weighted average; we can show that the measured noise is divided by $\sqrt{14/5}$ instead of $\sqrt{3}$ (3.5% overestimate) after the 2-average sequence.

| Average | Tsys | t | $f_{\rm res}$ | Theoretical noise | Measured noise | Status |
|---|---|---|---|---|---|---|
| 100+101+102 | 158.77 | 169.84 | 0.0195 | 0.087 | 0.127 | Unexpected[1] |
| 100+101 | 158.81 | 113.23 | 0.0195 | 0.107 | 0.154 | Unexpected[1] |
| (100+101)+102 | 122.99 | 169.84 | 0.0195 | 0.068 | 0.131 | Unexpected[2] |

Table 3: Same as Table 1, for the averaged spectra produced by the command `AVERAGE` in **CLASS** version mar21.

| Average | Tsys | t | $f_{res}$ | Theoretical noise | Measured noise | Status |
|---|---|---|---|---|---|---|
| 100+101+102 | 224.53 | 169.84 | 0.0195 | 0.123 | 0.127 | OK |
| 100+101 | 224.59 | 113.23 | 0.0195 | 0.151 | 0.154 | OK |
| (100+101)+102 | 224.53 | 169.84 | 0.0195 | 0.123 | 0.127 | OK |