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IRAM-30m EMIR time/sensitivity estimator

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Abstract

This memo describes the equations used in the IRAM-30m EMIR time/sensitivity estimator available in the `GILDAS/ASTRO` program. A large part of the memo aims at deriving sensitivity estimate for the case of On-The-Fly observations, which is not clearly documented elsewhere (to our knowledge). Numerical values of the different parameters used in the time/sensitivity estimator are grouped in appendix A.

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1 Generalities

1.1 The radiometer equation

The radiometer equation for a total power measurement reads

$$\sigma = \frac{T_{\rm sys}}{\eta_{\rm spec}\sqrt{d\nu\, t}}, \tag{1}$$

where σ is the rms noise obtained by integration during t in a frequency resolution $d\nu$ with a system whose system temperature is given by $T_{\rm sys}$ and spectrometer efficiency is $\eta_{\rm spec}$. However, total power measurement includes other contributions (*e.g.* the atmosphere emission) in addition to the astronomical signal. The usual way to remove most of the unwanted contributions is to switch, *i.e.* to measure alternatively on-source and off-source and then to subtract the off-source from the on-source measurements. It is easy to show that the rms noise of the obtained measurement is

$$\sigma = \sqrt{\sigma_{\rm on}^2 + \sigma_{\rm off}^2} = \frac{T_{\rm sys}}{\eta_{\rm spec}\sqrt{d\nu\, t_{\rm sig}}} \quad \text{with} \quad t_{\rm sig} = \frac{t_{\rm on}\, t_{\rm off}}{t_{\rm on} + t_{\rm off}}, \tag{2}$$

where $\sigma_{\rm on}$ and $\sigma_{\rm off}$ are the noise of the on and off measurement observed respectively during the $t_{\rm on}$ and $t_{\rm off}$ integration time. $t_{\rm sig}$ is just a useful intermediate quantity.

1.2 System temperature

The system temperature is a summary of the noise added by the system. This noise comes from 1) the receiver and the optics, 2) the emission of the sky, and 3) the emission picked up by the secondary side lobes of the telescope. It is usual to approximate it (in the $T_{\rm a}^\star$ scale) with

$$T_{\rm sys} = \frac{(1+G_{\rm im})\exp\{\tau_{\rm s}\, A\}}{F_{\rm eff}} \left[F_{\rm eff}\, T_{\rm atm}\, \left(1 - \exp\{-\tau_{\rm s}\, A\}\right) + (1-F_{\rm eff})\, T_{\rm cab} + T_{\rm rec}\right], \tag{3}$$

where $G_{\rm im}$ is the receiver image gain, $F_{\rm eff}$ the telescope forward efficiency, $A = 1/\sin(\text{elevation})$ the airmass, $\tau_{\rm s}$ the atmospheric opacity in the signal band, $T_{\rm atm}$ the mean physical atmospheric temperature, $T_{\rm cab}$ the ambient temperature in the receiver cabine and $T_{\rm rec}$ the noise equivalent temperature of the receiver and the optics. All those parameters are easily measured, except $\tau_{\rm s}$, which is depends on the amount of water vapor in the atmosphere and which is estimated by complex atmospheric models.

1.3 Elapsed telescope time

The goal of a time estimator is to find the elapsed telescope time ($t_{\rm tel}$) needed to obtain a given rms noise, while a sensitivity estimator aims at finding the rms noise obtained when observing during $t_{\rm tel}$. If $t_{\rm onoff}$ is the total integration time spent both on the on-source and off-source observations, then

$$t_{\rm onoff} = \eta_{\rm tel}\, t_{\rm tel}, \tag{4}$$

where $\eta_{\rm tel}$ is the efficiency of the observing mode, *i.e.* the time needed 1) to do calibrations (*e.g.* pointing, focus, temperature scale calibration), and 2) to slew the telescope between useful integrations.

The tuning of the receivers is not proportional to the total integration time but it should be added to the elapsed telescope time. A time estimator can hardly anticipate the total tuning time for a project. Indeed, one project (*e.g.* faint line detection) can request only one tuning to be used during many hours and another (*e.g.* line survey) can request a tuning every few minutes. In our case, we thus request that the estimator user add by hand the tuning time to the elapsed telescope time estimation.

1.4 The number of polarizations

Heterodyne mixers are coupled to a single linear polarization of the signal. Hence, heterodyne receivers have at least two mixers, each one sensitive to one of the two linear polarization of the incoming signal. Both mixers are looking at the same sky position. This implies that we have to distinguish between the time spent on a given position of sky and the human elapsed time. Indeed, we will use the time spent on a given position of the sky when estimating the sensitivity, while we will give human elapsed time for the telescope and the on and off times.

If the mixers are tuned at the same frequency, the times spent on and off in the same direction of the sky will be twice the human elapsed time. We thus have to introduce the number of polarization simultaneously tuned at the same frequency, n_{pol}, which can be set to 1 or 2. It happens that for EMIR, the two polarizations are always tuned at the same frequency, *i.e.* $n_{\text{pol}} = 2$. The simplest way to take into account the distinction between human time and sky time is to slightly modify the radiometer equation to take into account the number of polarization

$$\sigma = \frac{T_{\text{sys}}}{\eta_{\text{spec}} \sqrt{d\nu \, n_{\text{pol}} \, t_{\text{sig}}}} \quad \text{with} \quad t_{\text{sig}} = \frac{t_{\text{on}} \, t_{\text{off}}}{t_{\text{on}} + t_{\text{off}}}. \tag{5}$$

This equation implies that t_{on}, t_{off}, t_{onoff} and t_{tel} will be human times.

1.5 Switching modes and observation kinds

Switching is done in two main ways.

Position switch where the off-measurement is done on a close-by sky position devoid of signal. Wobbler switching is a particular case.

Frequency switch where the telescope always points towards the the source and the switching is done in the frequency (velocity) space.

Moreover, there are two main observation kinds.

Tracked observations where the telescope track the source, *i.e.* it always observes the same position in the source referential. The result is a single spectra.

On-The-Fly observations where the telescope continuously slew through the source with time to map it. The result is a cube of spectra.

In the following, we will work out the equations needed by the time/sensitivity estimator for each combination.

2 Tracked observations

2.1 Frequency switched

In this case, all the time is spent in the direction of the source. However, the frequency switching also implies that all this times can be counted as on-source and off-source times. Thus

$$t_{\text{onoff}} = t_{\text{on}} = t_{\text{off}}, \tag{6}$$

$$t_{\text{sig}} = \frac{t_{\text{on}}}{2} = \frac{t_{\text{off}}}{2} = \frac{t_{\text{onoff}}}{2}, \tag{7}$$

and

$$\sigma_{\text{fsw}} = \frac{\sqrt{2}\, T_{\text{sys}}}{\eta_{\text{spec}} \sqrt{d\nu \, n_{\text{pol}} \, \eta_{\text{tel}} \, t_{\text{tel}}}}. \tag{8}$$

2.2 Position switched

In this case, only half of the time is spent in the direction of the source. Thus

$$t_{\rm on} = t_{\rm off} = \frac{t_{\rm onoff}}{2}, \tag{9}$$

$$t_{\rm sig} = \frac{t_{\rm on}}{2} = \frac{t_{\rm off}}{2} = \frac{t_{\rm onoff}}{4}, \tag{10}$$

and

$$\sigma_{\rm psw} = \frac{2\,T_{\rm sys}}{\eta_{\rm spec}\sqrt{d\nu\, n_{\rm pol}\,\eta_{\rm tel}\,t_{\rm tel}}}. \tag{11}$$

2.3 Comparison

For tracked observations, position switched observations results in a noise rms $\sqrt{2}$ larger than frequency switched observations for the same elapsed telescope time. In other words, frequency switched observations are twice as efficient in time to reach the same rms noise than position switched observations.

However, time efficiency is not the only criteria of choice. Indeed, with the current generation of receivers (before march 2009), the IF bandpass is much cleaner in position switched than in frequency switched observations. Frequency switched is thus really useful only when the lines are narrow so that the IF bandpass can be easily cleaned out through baselining with low order polynomials.

3 On-The-Fly observations

3.1 Additional notions and notations

The On-The-Fly (OTF) observing mode is used to map a given region of the sky. The time/sensitivity estimator will have to link the elapsed telescope time to cover the whole mapped region to the sensitivity in each independent resolution element. To do this, we need to introduce

- $A_{\rm map}$ and $A_{\rm beam}$, which are respectively the area of the map and the area of the resolution element. The map area is a user input while the resolution area is linked to the telescope full width at half maximum (θ) by

$$A_{\rm beam} = \frac{\eta_{\rm grid}\,\pi\,\theta^2}{4\,\ln(2)} \tag{12}$$

  where $\eta_{\rm grid}$ comes from the fact that the OTF data is gridded by convolution. When the convolution kernel is a Gaussian of FWHM equal to $\theta/3$ (the default inside the `GILDAS/CLASS` software), it is easy to show that

$$\eta_{\rm grid} = 1 + \frac{1}{9} \simeq 1.11. \tag{13}$$

- The number of independent measurement ($n_{\rm beam}$) in the final map which is given by

$$n_{\rm beam} = \frac{A_{\rm map}}{A_{\rm beam}}. \tag{14}$$

- The on and off time spent per independent measurement, $t_{\rm on}^{\rm beam}$ and $t_{\rm off}^{\rm beam}$. The associated $t_{\rm sig}^{\rm beam}$ can then be written

$$t_{\rm sig}^{\rm beam} = \frac{t_{\rm on}^{\rm beam}\,t_{\rm off}^{\rm beam}}{t_{\rm on}^{\rm beam} + t_{\rm off}^{\rm beam}} \tag{15}$$

- The on and off time spent to map the whole map, $t_{\rm on}^{\rm tot}$ and $t_{\rm off}^{\rm tot}$. $t_{\rm onoff}$ is deduced from $t_{\rm on}^{\rm tot}$ and $t_{\rm off}^{\rm tot}$ in a way which depends on the switching scheme.

In addition, we must ensure that the user does not try to scan faster than the telescope can slew. To do this, we need to introduce

- The linear scanning speed, v_{linear}, and its maximum value, $v_{\text{linear}}^{\text{max}}$.
- The area scanning speed, v_{area}, and its maximum value, $v_{\text{area}}^{\text{max}}$. When the scanning pattern is linear, then v_{area} and v_{linear} are linked through

$$v_{\text{area}} = v_{\text{linear}}\,\Delta\theta, \tag{16}$$

where $\Delta\theta$ is the separation between consecutive rows. To avoid nasty signal and noise aliasing problems, we must ensure a Nyquist sampling, *i.e.*

$$\Delta\theta = \frac{\theta}{2.5}. \tag{17}$$

3.2 Frequency switched

In frequency switched observations, the switching happens as the telescope is slewed. This is correct as long as the switching time is much smaller than the time needed to slew a significant fraction of the telescope beam.

It is easy to understand that

$$t_{\text{onoff}} = t_{\text{on}}^{\text{tot}} = t_{\text{off}}^{\text{tot}}, \tag{18}$$

$$t_{\text{on}}^{\text{beam}} = t_{\text{off}}^{\text{beam}} = \frac{t_{\text{onoff}}}{n_{\text{beam}}}, \tag{19}$$

$$t_{\text{sig}}^{\text{beam}} = \frac{t_{\text{on}}^{\text{beam}}}{2} = \frac{t_{\text{off}}^{\text{beam}}}{2} = \frac{t_{\text{onoff}}}{2n_{\text{beam}}}, \tag{20}$$

and

$$\sigma_{\text{fsw}} = \frac{\sqrt{2\,n_{\text{beam}}}\,T_{\text{sys}}}{\eta_{\text{spec}}\sqrt{d\nu\,n_{\text{pol}}\,\eta_{\text{tel}}\,t_{\text{tel}}}}. \tag{21}$$

The velocity check can then be written as

$$\frac{A_{\text{map}}}{t_{\text{onoff}}} \leq v_{\text{area}}^{\text{max}}. \tag{22}$$

3.3 Position switched

3.3.1 Two key points: 1) Sharing OFF among many ONs and 2) system stability timescale

When the stability of the system is long enough, we can share the same off for several *independent* on-positions measured in a row (*e.g.* ON-ON-ON-OFF-ON-ON-ON-OFF...). The first key point here is the fact that the on-positions must be independent. The OTF is an observing mode where the sharing of the off can be used because the goal is to map a given region of the sky made of independent positions or resolution elements. When sharing the off-position between several on, Ball (1976) showed that the optimal off integration time is

$$t_{\text{off}}^{\text{optimal}} = \sqrt{n_{\text{on/off}}}\,t_{\text{on}} \tag{23}$$

where $n_{\text{on/off}}$ is the number of on measurements per off. Replacing t_{off} by its optimal value in eq. 5, we obtain

$$t_{\text{sig}} = \frac{t_{\text{on}}}{1+\frac{1}{\sqrt{n_{\text{on/off}}}}} \quad \text{and} \quad \sigma = \frac{T_{\text{sys}}}{\eta_{\text{spec}}\sqrt{d\nu\,n_{\text{pol}}\,t_{\text{on}}}}\sqrt{1+\frac{1}{\sqrt{n_{\text{on/off}}}}}. \tag{24}$$

We thus see that the rms noise decreases when the number of independent on per off increases. It seems tempting to have only one off for all the on positions of the OTF map. However, the second key point of the method is that the system must be stable between the first and last on measurement. To take this point into account we must introduce

- The concept of submap, which is a part of a map observed between two successive off measurements.
- $A_{\rm submap}$, which is the area covered by the telescope in each submap.
- $n_{\rm submap}$ the number of such submaps needed to cover the whole map area.
- $t_{\rm stable}$, the typical time where the system is stable. This time will be the maximum time between two off measurements, which is noted $t_{\rm submap}$.
- $n_{\rm cover}$, the number of coverages needed either to reach a given sensitivity or to exhaust the acquisition time.
- $t_{\rm on}^{\rm cover}$ and $t_{\rm off}^{\rm cover}$ are the times spent respectively on and off per independent measurement and per coverage.

We note that the number of on per off ($n_{\rm on/off}$) is a purely geometrical quantity. This implies that the time spent off is linked to the time spent on by Eq. 23 both in each individual coverage and when averaging all the coverages.

3.3.2 Relation between $t_{\rm onoff}$ and $t_{\rm sig}^{\rm beam}$

By construction

- The number of submaps is the area of the map divided by the area of a submap
$$n_{\rm submap} = \frac{A_{\rm map}}{A_{\rm submap}}. \tag{25}$$
- The number of on per off is the number of independent resolution elements in each submap
$$n_{\rm on/off} = \frac{A_{\rm submap}}{A_{\rm beam}}. \tag{26}$$
- The number of independent resolution elements in the map is the product of number of submaps by the number of on per off
$$n_{\rm beam} = n_{\rm submap}\, n_{\rm on/off}. \tag{27}$$
- The submap area is the product of the area velocity by the time to cover it
$$A_{\rm submap} = v_{\rm area}\, t_{\rm submap}. \tag{28}$$
- The time to scan a submap is the sum of the $n_{\rm on/off}$ independent on integration time
$$t_{\rm submap} = n_{\rm on/off}\, t_{\rm on}^{\rm cover}. \tag{29}$$
- The relations between times per coverage and times integrated over all the coverages are
$$t_{\rm on}^{\rm beam} = n_{\rm cover}\, t_{\rm on}^{\rm cover} \quad \text{and} \quad t_{\rm off}^{\rm beam} = n_{\rm cover}\, t_{\rm off}^{\rm cover} \quad \text{with} \quad t_{\rm off}^{\rm cover} = \sqrt{n_{\rm on/off}}\, t_{\rm on}^{\rm cover}. \tag{30}$$
- Using the last two points, it is easy to derive
$$t_{\rm sig}^{\rm beam} = n_{\rm cover}\, t_{\rm sig}^{\rm cover} = \frac{n_{\rm cover}\, t_{\rm submap}}{n_{\rm on/off} + \sqrt{n_{\rm on/off}}}. \tag{31}$$

- Finally, the total time spent on and off is given by

$$t_{\rm onoff} = n_{\rm cover}\, n_{\rm submap}\, (n_{\rm on/off}\, t_{\rm on}^{\rm cover} + t_{\rm off}^{\rm cover}). \tag{32}$$

Using Eqs. 23 and 29, we derive

$$t_{\rm onoff} = n_{\rm cover}\, t_{\rm submap}\, n_{\rm submap} \left(1 + \frac{1}{\sqrt{n_{\rm on/off}}}\right). \tag{33}$$

Both $t_{\rm sig}^{\rm beam}$ and $t_{\rm onoff}$ are proportional to $n_{\rm cover}\, t_{\rm submap}$ (cf. Eqs. 31 and 33). It is thus easy to derive that

$$t_{\rm onoff} = t_{\rm sig}^{\rm beam}\, n_{\rm submap} \left(1 + \sqrt{n_{\rm on/off}}\right)^2. \tag{34}$$

Using Eq. 27, we can replace $n_{\rm on/off}$ and obtain

$$t_{\rm onoff} = t_{\rm sig}^{\rm beam} \left(\sqrt{n_{\rm submap}} + \sqrt{n_{\rm beam}}\right)^2. \tag{35}$$

Using Eqs. 5, 4 and 35, we obtain

$$\sigma_{\rm psw} = \frac{\left(\sqrt{n_{\rm beam}} + \sqrt{n_{\rm submap}}\right) T_{\rm sys}}{\eta_{\rm spec} \sqrt{d\nu\, n_{\rm pol}\, \eta_{\rm tel}\, t_{\rm tel}}}. \tag{36}$$

The last equation in theory enables us to find the rms noise as a function of the elasped telescope time (sensitivity estimation) and vice-versa (time estimation). However, it is not fully straightforward because we must enforce that $n_{\rm cover}$ and $n_{\rm submap}$ have an integer value.

3.3.3 Time/Sensitivity estimation

This paragraph describes the algorithm to do a time/sensitivity estimation for a position-switched On-The-Fly observation.

Step #1: Computation of $n_{\rm beam}$ and $n_{\rm submap}$
$n_{\rm beam}$ is just computed as the ratio $A_{\rm map}/A_{\rm beam}$. Using Eqs. 28 and 25, we obtain

$$n_{\rm submap} = \frac{A_{\rm map}}{v_{\rm area}\, t_{\rm submap}}. \tag{37}$$

Using this equation, we start to compute $n_{\rm submap}$ for $t_{\rm submap} = t_{\rm stable}$ and $v_{\rm area} = v_{\rm area}^{\rm max}$. We want to enforce the integer character of $n_{\rm submap}$ in a way which decreases the product $t_{\rm submap}\, v_{\rm area}$. To do this, we use

$$n_{\rm submap} = 1 + \texttt{int}(n_{\rm submap}). \tag{38}$$

Eq. 38 ensures that $t_{\rm submap}\, v_{\rm area} < t_{\rm stable}\, v_{\rm area}^{\rm max}$. The value of $v_{\rm area}$ must be decreased so that Eq. 37 is enforced.

Step #2: Computation of $t_{\rm tel}$ or σ We use the following equations in descending order to compute the elapsed telescope time and in ascending order to compute the rms noise level:

1. $$t_{\rm sig}^{\rm beam} = \frac{T_{\rm sys}^2}{\eta_{\rm spec}^2\, \sigma^2\, d\nu\, n_{\rm pol}}, \tag{39}$$
2. $$t_{\rm onoff} = t_{\rm sig}^{\rm beam} \left(\sqrt{n_{\rm submap}} + \sqrt{n_{\rm beam}}\right)^2, \tag{40}$$
3. $$\eta_{\rm tel}\, t_{\rm tel} = t_{\rm onoff}. \tag{41}$$

Step #3: Computation of derived quantities

$$1. \quad n_{\text{on/off}} = \frac{n_{\text{beam}}}{n_{\text{submap}}}, \tag{42}$$

$$2. \quad n_{\text{cover}} = \frac{t_{\text{sig}}^{\text{beam}}}{t_{\text{submap}}} \left(n_{\text{on/off}} + \sqrt{n_{\text{on/off}}} \right), \tag{43}$$

$$3. \quad t_{\text{on}}^{\text{beam}} = \frac{n_{\text{cover}}\, t_{\text{submap}}}{n_{\text{on/off}}} \tag{44}$$

$$3. \quad t_{\text{off}}^{\text{beam}} = t_{\text{on}}^{\text{beam}} \sqrt{n_{\text{on/off}}}. \tag{45}$$

Step #4: Is n_{cover} an integer number? The interpretation of the above equation to compute n_{cover} has two cases.

1. $n_{\text{cover}} < 1$. This means that either the user tries to cover a too large sky area in the given telescope elasped time (sensitivity estimation) or the telescope need a minimum time to cover A_{map} at the maximum velocity possible with the telescope and this minimum time implies a more sensitive observation than requested by the user (time estimation).
2. $n_{\text{cover}} \geq 1$. n_{cover} will generally not be an integer, we can think to decrease t_{submap} from t_{stable} to obtain an integer value. However, this must be done at constant $A_{\text{submap}}(= v_{\text{area}}\, t_{\text{submap}})$. Decreasing t_{submap} thus implies increasing v_{area}. It is not clear that this is possible because of the constraint $v_{\text{area}} < v_{\text{area}}^{\text{max}}$. Another way to deal with this is to keep t_{submap} to its maximum value and to adjust t_{tel} (sensitivity estimation) or t_{sig} and thus σ (time estimation) to obtain an integer value of n_{cover}. However, this implies a change in the wishes of the user. The program can not make such a decision alone and we will only warn the user. Indeed, the worst case is when n_{cover} is changing from 1 to 2 because this can decrease the sensitivity by a factor 1.4 (sensitivity estimation) or double the elapsed telescope time (time estimation). The larger the value of n_{cover} the less harm it is to enforce the integer character of n_{cover}.

3.4 Comparison

Contrary to tracked observations, the position switched observing mode can be more efficient than the frequency switched observing mode. Indeed, in frequency switch, the same time is spent in the on and off spectrum. When subtracting them, the off brings as much noise as the on. In position switch, the same off can be shared between many ons, in which case the optimal integration time on the off is much larger than on each independent on spectrum. Hence, the noise brought by the off spectrum can be much smaller than the noise brought by the on spectrum.

For frequency switched observations,

$$\sigma_{\text{fsw}} = \frac{\sqrt{2\, n_{\text{beam}}}\, T_{\text{sys}}}{\eta_{\text{spec}} \sqrt{d\nu\, n_{\text{pol}}\, \eta_{\text{tel}}\, t_{\text{tel}}}}, \tag{46}$$

while for position switched observations,

$$\sigma_{\text{psw}} = \frac{\left(\sqrt{n_{\text{beam}}} + \sqrt{n_{\text{submap}}} \right)\, T_{\text{sys}}}{\eta_{\text{spec}} \sqrt{d\nu\, n_{\text{pol}}\, \eta_{\text{tel}}\, t_{\text{tel}}}}. \tag{47}$$

We thus have

$$\frac{\sigma_{\text{psw}}}{\sigma_{\text{fsw}}} = \frac{1}{\sqrt{2}} \left(1 + \sqrt{\frac{n_{\text{submap}}}{n_{\text{beam}}}} \right). \tag{48}$$

Position switched OTF is more efficient than frequency switched OTF for

$$\frac{n_{\text{beam}}}{n_{\text{submap}}} = n_{\text{on/off}} \geq \frac{1}{3 - 2\sqrt{2}} \sim 6. \tag{49}$$

Moreover, $\sigma_{\rm psw}/\sigma_{\rm fsw} \simeq 0.84$ for $n_{\rm on/off} = 30$, and $\simeq 0.78$ for $n_{\rm on/off} = 100$. Using eqs. 28 and 26, we see that the limit on the maximum number of on per off is set by

$$n_{\rm on/off} = \frac{t_{\rm stable}}{A_{\rm beam}/v_{\rm area}^{\rm max}}, \tag{50}$$

i.e. the ratio of the maximum system stability time by the minimum time required to map a telescope beam.

As for tracked observations, there are other considerations to be taken into account. For extra-galactic observations, the lines are large which excludes the use of frequency switched observations. For Galactic observations, the intrinsic sensitivity of the receivers may make it difficult to find a closeby position devoid of signal. We can still use the position switched OTF observing mode. But we then have to observe the off position in frequency switched track observing mode long enough to be able to add back the off astronomical signal.

4 Acknowledgement

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References

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A Numerical values

This appendix groups all the numerical values used in the time/sensitivity estimator. We made conservative choices for two reasons: 1) time/sensitivity estimators tend to be too optimistics and 2) EMIR is a new generation of receivers which had not yet been tested at the telescope.

A.1 Overheads

- $\eta_{\text{tel}} = 0.5$.
- After estimating the number of tunings needed to complete the project, the user has to add to the telescope time 30 minutes per tuning (this includes the observation of a line calibrator).

A.2 Atmosphere

- $T_{\text{atm}} = 250$ K.
- The opacities at signal frequencies are computed with a recent version of the ATM program (maintained by J.R. Pardo).
- They are computed for 3 different amount of water vapor per season (1, 2 and 4 mm for the winter season and 2, 4 and 7 mm for the summer season).

A.3 Telescope

- $T_{\text{cab}} = 290$ K.
- $F_{\text{eff}} = 0.95$ at 3 mm, 0.93 at 2 mm, 0.91 at 1 mm and 0.88 at 0.8 mm.

A.4 Frontends

Warning: Please do not quote these values in your papers. You should refer to the publications which fully describe the receivers.

- The receiver temperature is the sum of
  - The mixer temperature: Typically 50 K below 260 GHz and 70 K above;
  - The mirror losses: Typically 10 K;
  - The dichroic losses: Typically 15 K. Nota Bene: Dichroics enable dual frequency observation by frequency separation of the sky signal.

  We end up with $T_{\text{rec}} = 75$ K below 260 GHz and $Trec = 95$ K above 260 GHz.
- $G_{\text{im}} = 0.1$.

A.5 Backends

- $\eta_{\text{spec}} = 0.87$ because of the 2 bit quantization at the input of the correlators.
- The noise equivalent bandwidth of our correlators is almost equal to the channel spacing. So we do not take this into account in our estimation.

A.6 On-The-Fly

- $t_{\text{stable}} = 2$ minutes.
- $\theta = \frac{2460''}{\nu \, \text{GHz}}$.
- The maximum linear velocity is limited by the maximum dumping rate of $f_{\text{dump}} = 2$ Hz. We know that in order to avoid beam elongation along the scanning direction, we need to sample at least 4 points per beam in the scanning direction. We thus end up with

$$v_{\text{linear}}^{\text{max}} = f_{\text{dump}} \, \frac{\theta}{4} \, \text{arcsec/s} \tag{51}$$

and

$$v_{\text{area}}^{\text{max}} = f_{\text{dump}} \, \frac{\theta}{2.5} \, \frac{\theta}{4} \, \text{arcsec}^2/\text{s} \quad \text{or} \quad v_{\text{area}}^{\text{max}} = f_{\text{dump}} \, \frac{\theta^2}{10} \, \text{arcsec}^2/\text{s} \tag{52}$$

B Optimal number of ON per OFF measurements

This section is just a reformulation of the original demonstration by Ball (1976).

Let's assume that we are measuring $n_{\mathrm{on/off}}$ *independent* on-positions for a single off. The same integration time (t_{on}) is spent on each on-position and the off integration time is

$$t_{\mathrm{off}} = \alpha\, t_{\mathrm{on}}, \tag{53}$$

where α can be varied. Using eq. 2 and $t_{\mathrm{onoff}} = n_{\mathrm{on/off}} t_{\mathrm{on}} + t_{\mathrm{off}} = (n_{\mathrm{on/off}} + \alpha)\, t_{\mathrm{on}}$, it can be shown than

$$t_{\mathrm{onoff}} = \frac{T_{\mathrm{sys}}^2}{\eta_{\mathrm{spec}}^2\, \sigma^2\, d\nu} \left(1 + n_{\mathrm{on/off}} + \alpha + \frac{n_{\mathrm{on/off}}}{\alpha}\right). \tag{54}$$

Differenciating with respect to α, we obtain

$$\frac{dt_{\mathrm{onoff}}}{d\alpha} \propto 1 - \frac{n_{\mathrm{on/off}}}{\alpha^2} \tag{55}$$

Setting the result to zero then gives that the minimum elapsed time to reach a given rms noise is obtained for

$$\alpha = \sqrt{n_{\mathrm{on/off}}} \quad \text{or} \quad t_{\mathrm{off}}^{\mathrm{optimal}} = \sqrt{n_{\mathrm{on/off}}}\, t_{\mathrm{on}}. \tag{56}$$